Computational Challenges from Imaging X-ray Polarimetry

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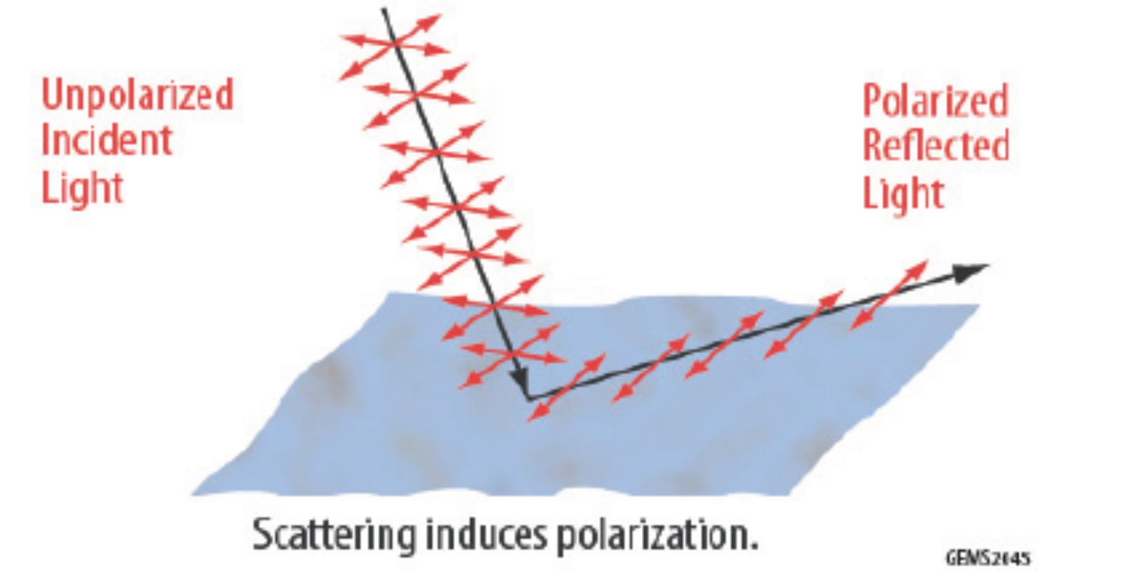
and the IXPE Team

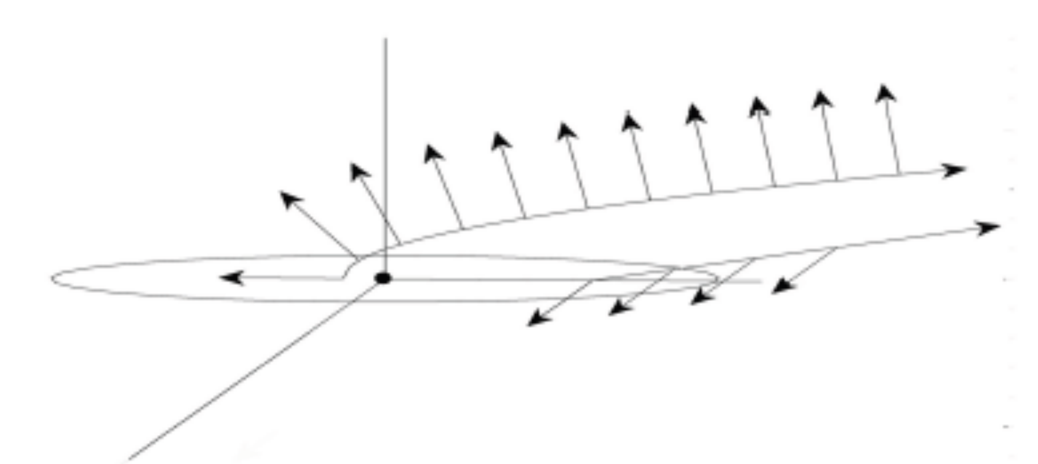
Outline

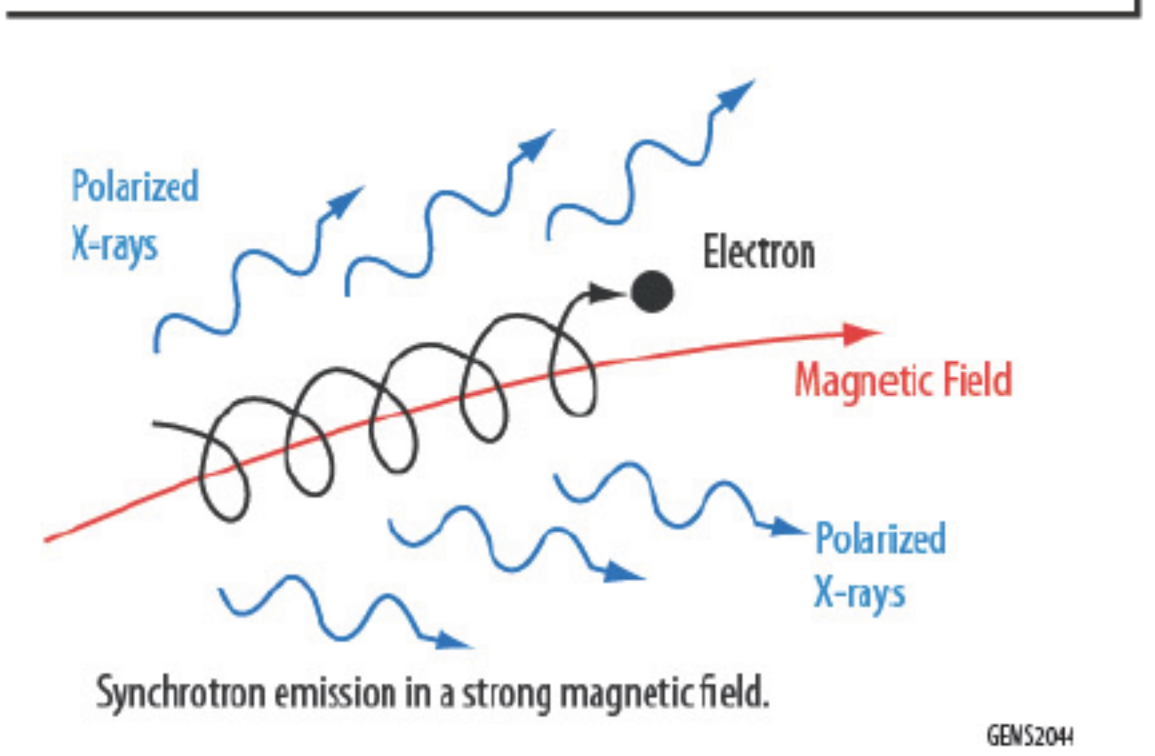
- Introduction to Polarimetry
- IXPE: the Imaging X-ray Polarization Explorer
- Computational Challenges
  - Basic measurements
  - Event track measurement — Machine Learning?
  - Modeling in 7 dimensions (E, t, α, δ, I, Q, U) — nonparametric Bayesian priors?
  - Testing models on event lists — nearest neighbor testing, Approximate Bayesian Computation?

Polarimetry Probes of Physics

- Polarization measurements allow us to study:
  - ✓ Scattering
  - ✓ Magnetic fields
  - ✓ Strong gravity





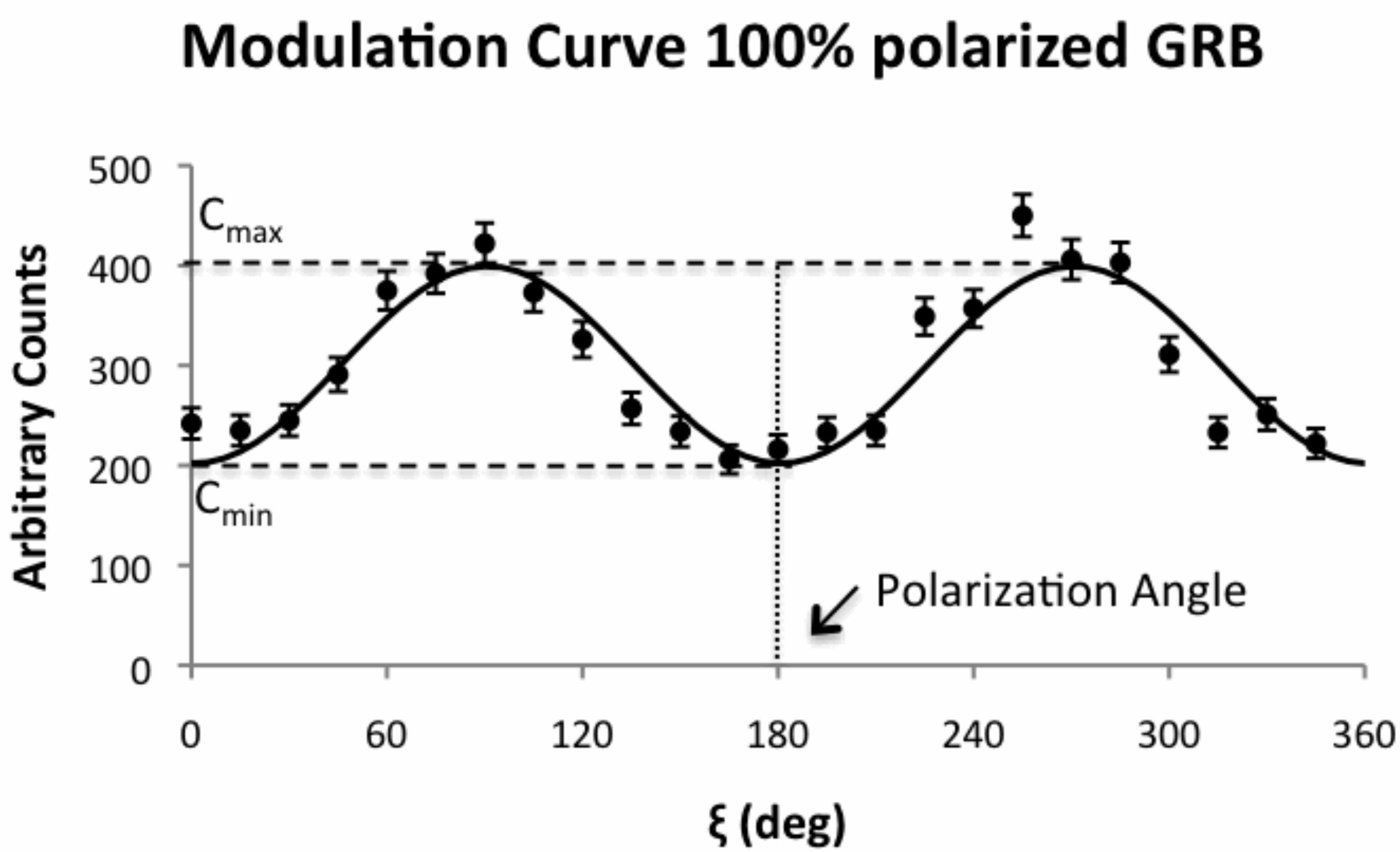


Basics of Polarized Light

- All light waves are polarized
- Stokes parameters are handy:
  - I = total intensity
  - Q, U are orthogonal linearly polarized parts
  - V is circular (+ or -) polarized intensity
- Common alternative: Π, ϕ
  - $\Pi = (Q^2 + U^2)^{1/2} / I$
  - $\phi = \tan^{-1}(Q/U)$
- A beam is “unpolarized” if the photon **set** is randomly polarized ($\Pi = V = 0$)
- MDP = ‘Minimum Detectable Polarization’ (99% conf.): $3.035\sqrt{2}\frac{\sqrt{C_S + C_B}}{C_S\langle\mu\rangle}$
- All photons also have energy (E), time (t), sky position (α,δ)

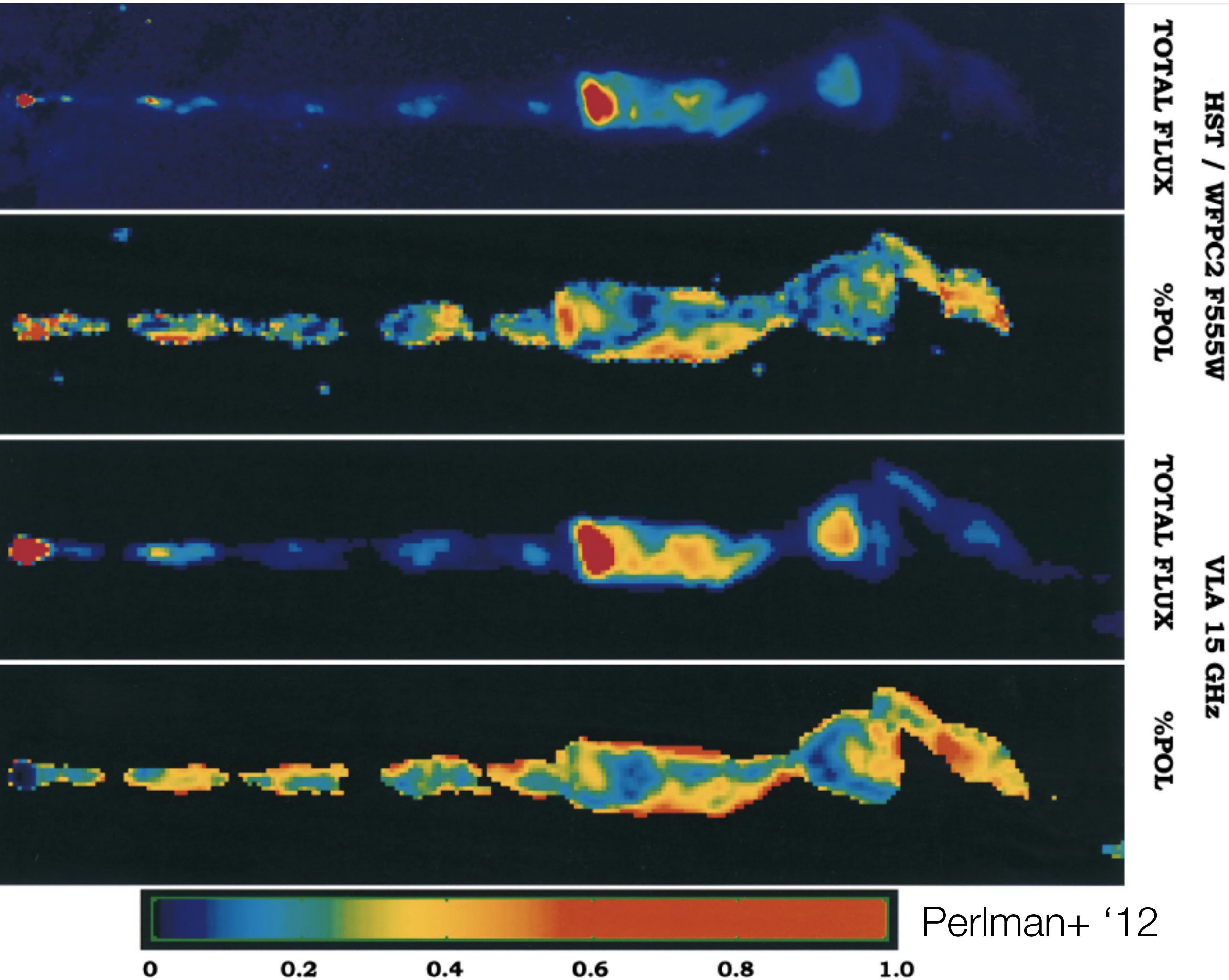
Modulation of Polarized Signals

Modulation Factor = $\mu = (C_{max}-C_{min})/(C_{max}+C_{min})$

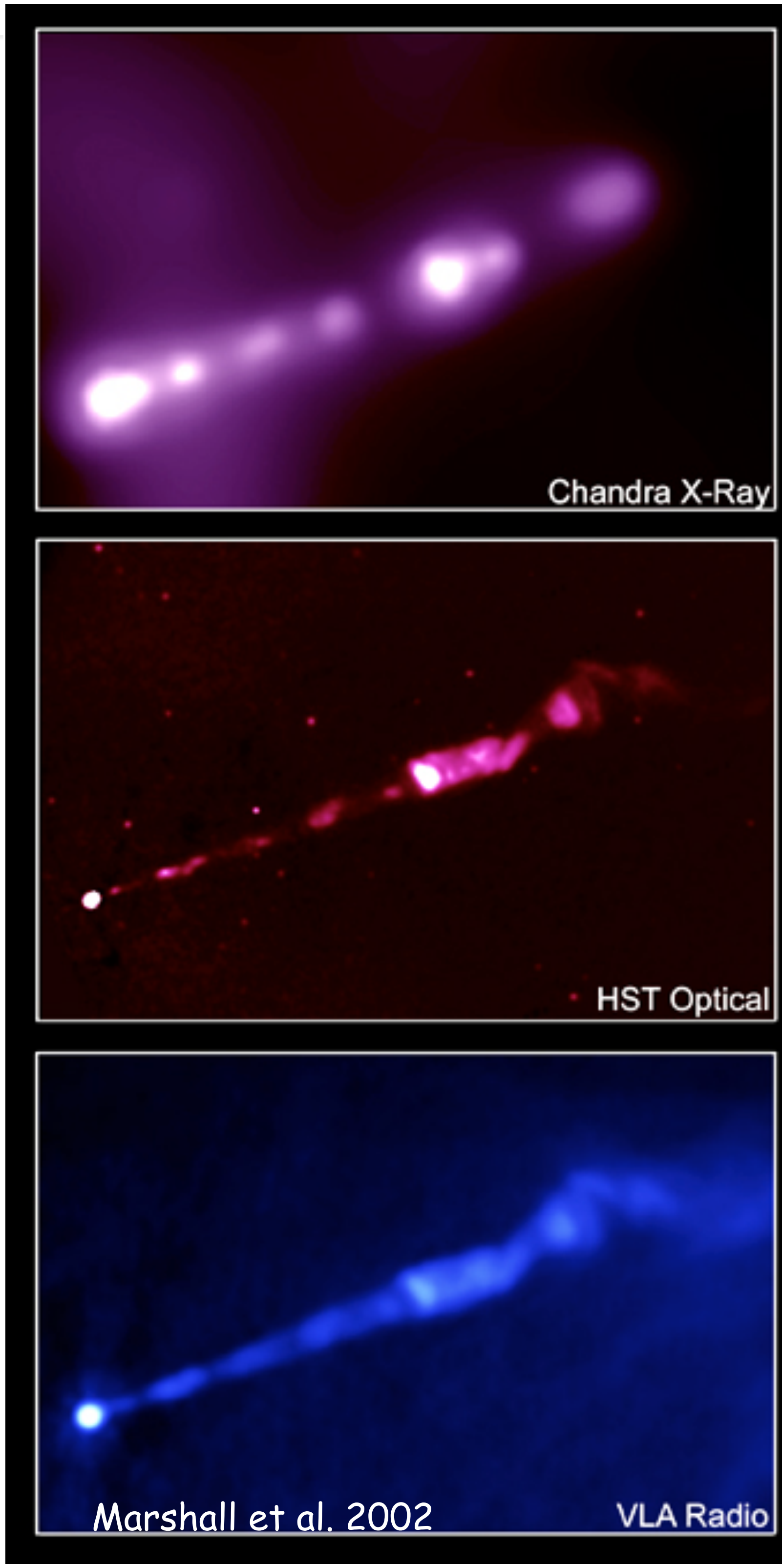


http://www.isdc.unige.ch/polar/modfactor

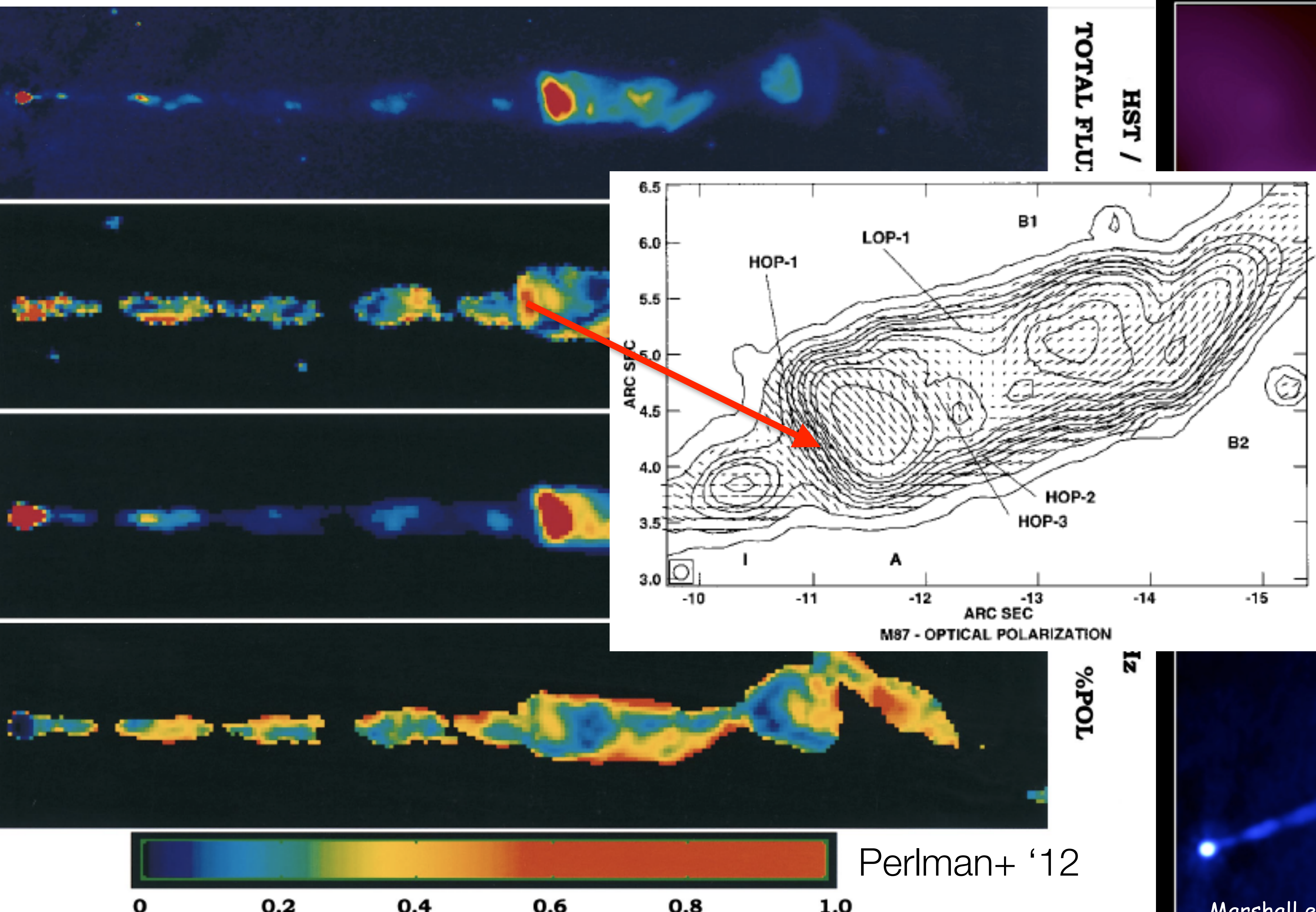
AGN Jet Polarimetry (M 87)



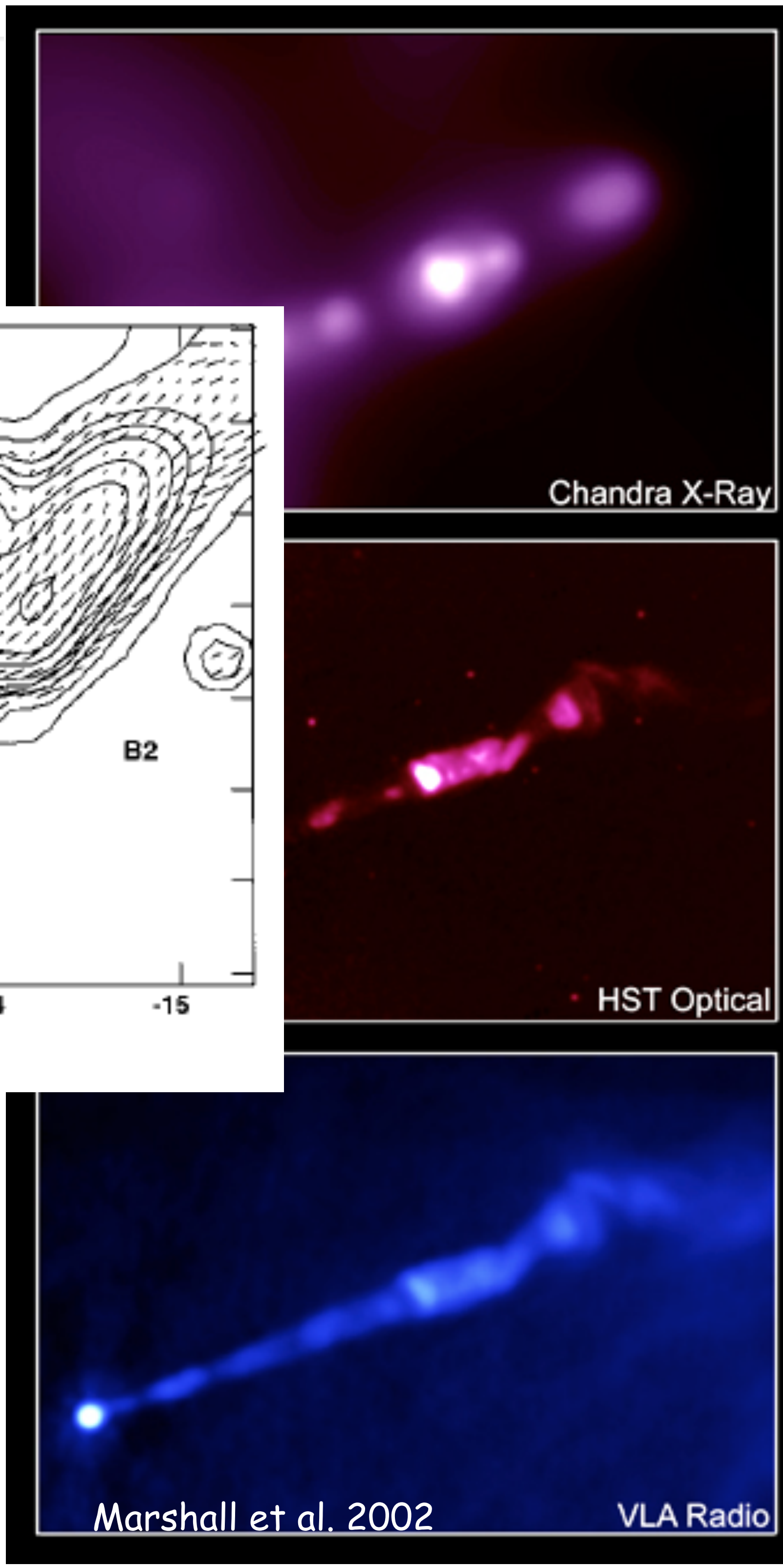
Perlman+ ‘12



AGN Jet Polarimetry (M 87)



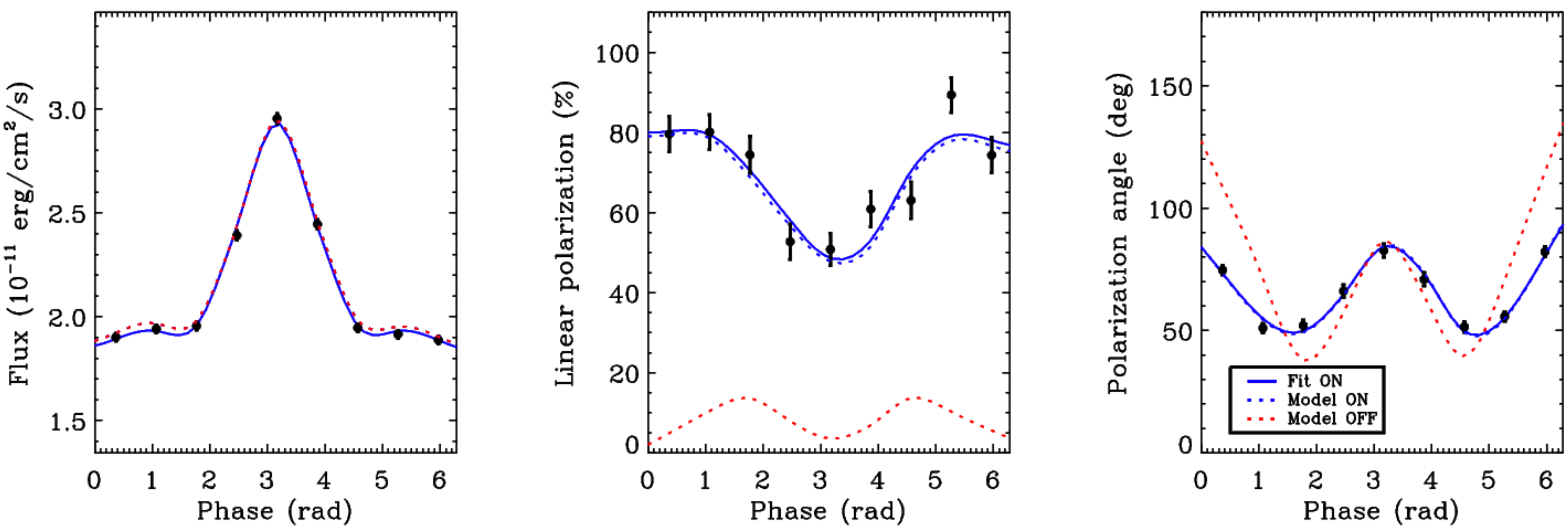
Perlman+ '12



Imaging Polarimetry

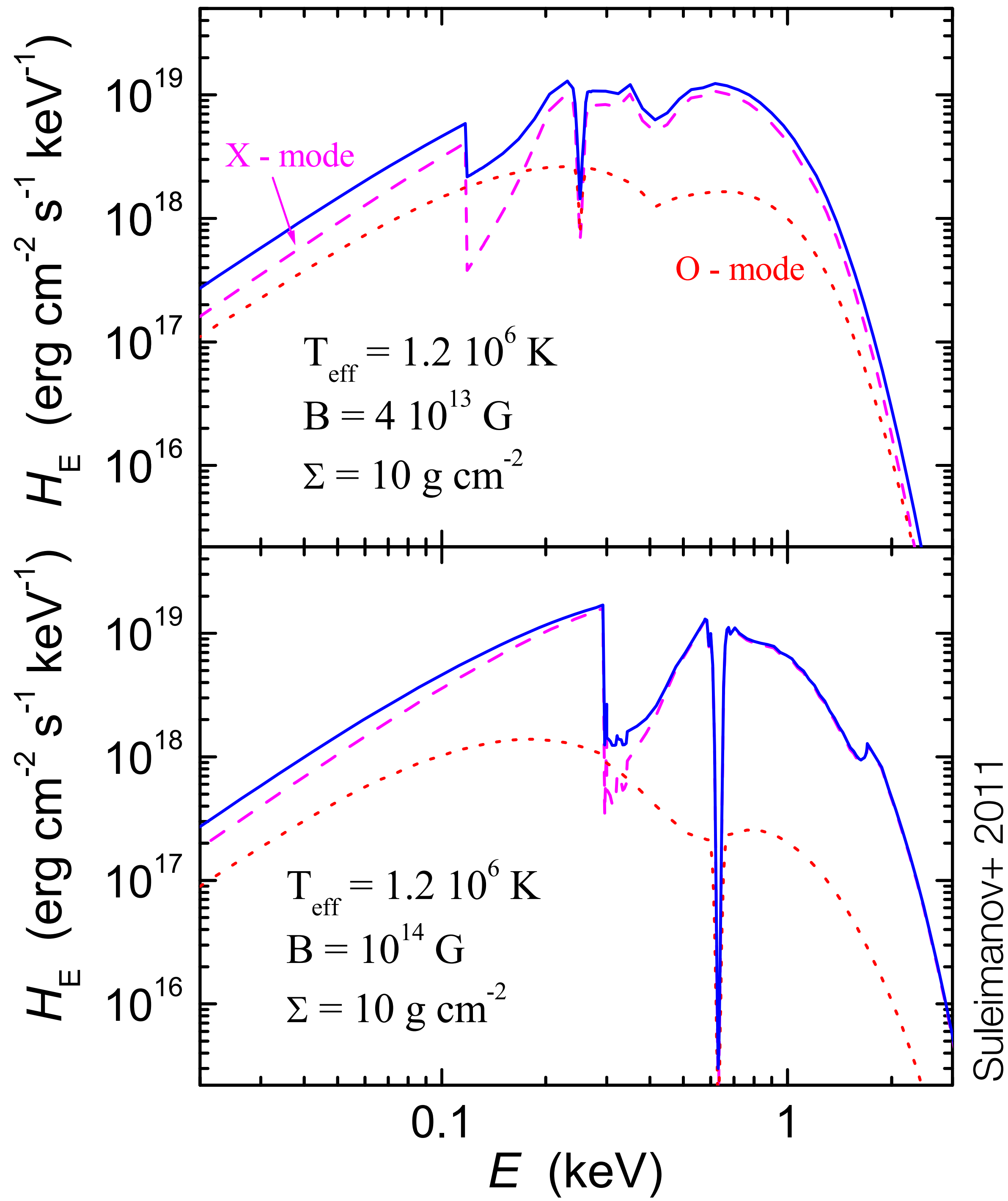
Testing Quantum Electrodynamics with Magnetars

- Magnetars: slowly rotating neutron stars with B > 10^{14} G
- Magnetized vacuum is birefringent
- Flux is unaffected but polarization fraction and angle change with spin phase

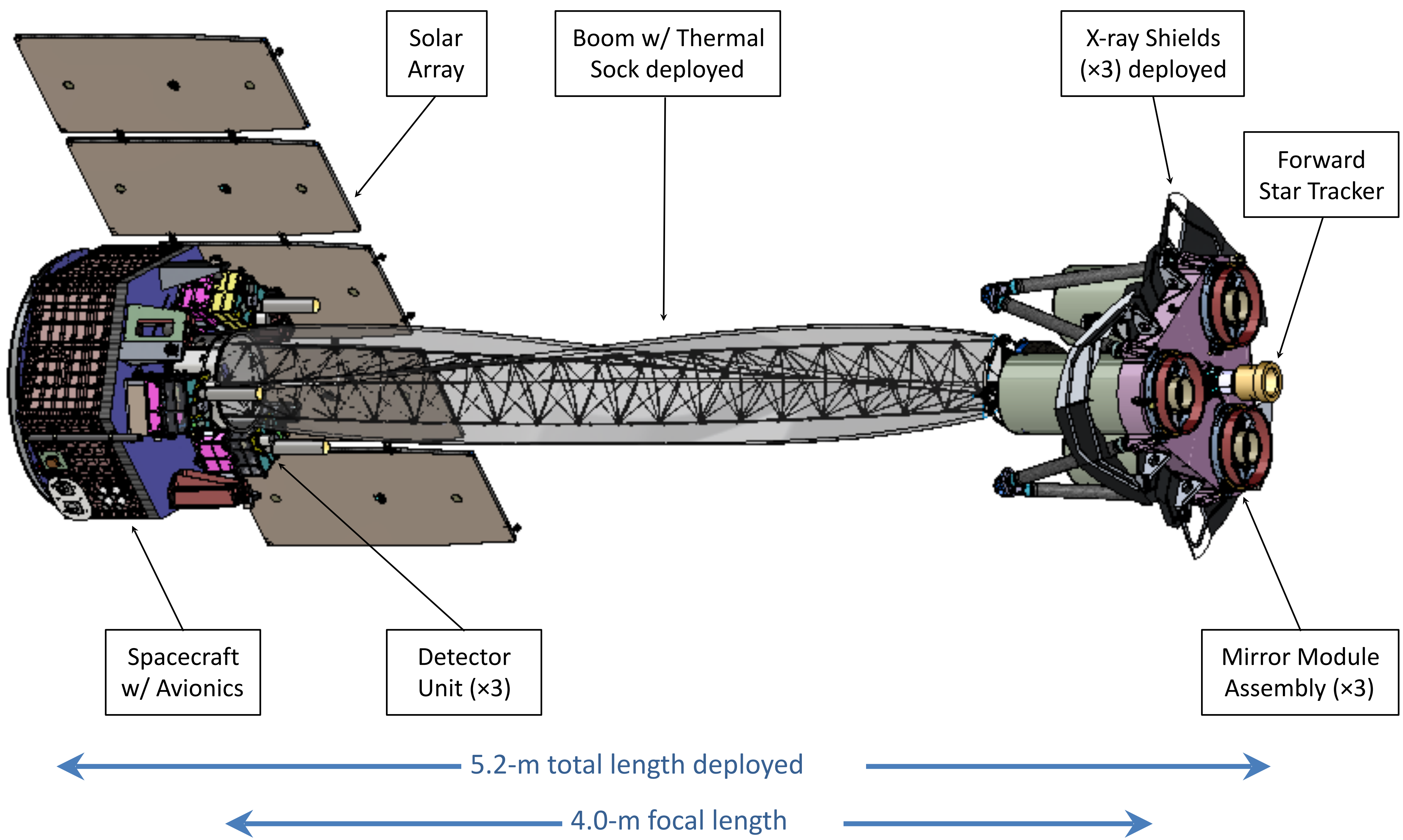


A Science Goal: Neutron Star Atmospheres

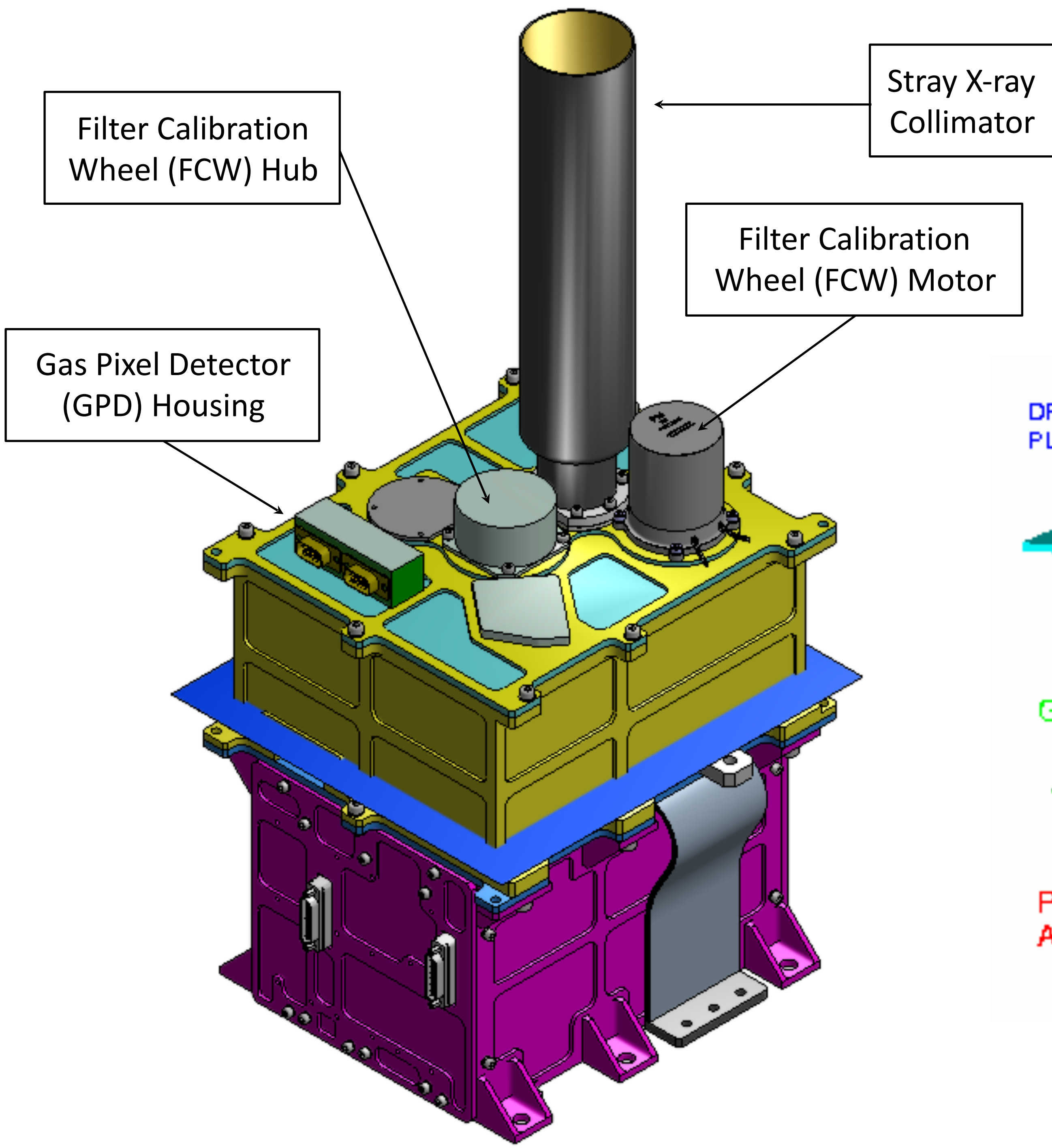
- Π and ϕ depend on B-field direction and N-star orientation (pulse phase)
- Atmospheres show features now found in spectra of isolated N-stars
- Polarization data would distinguish features in spectra
- Atmosphere models are used to determine R^2, g to give M,R

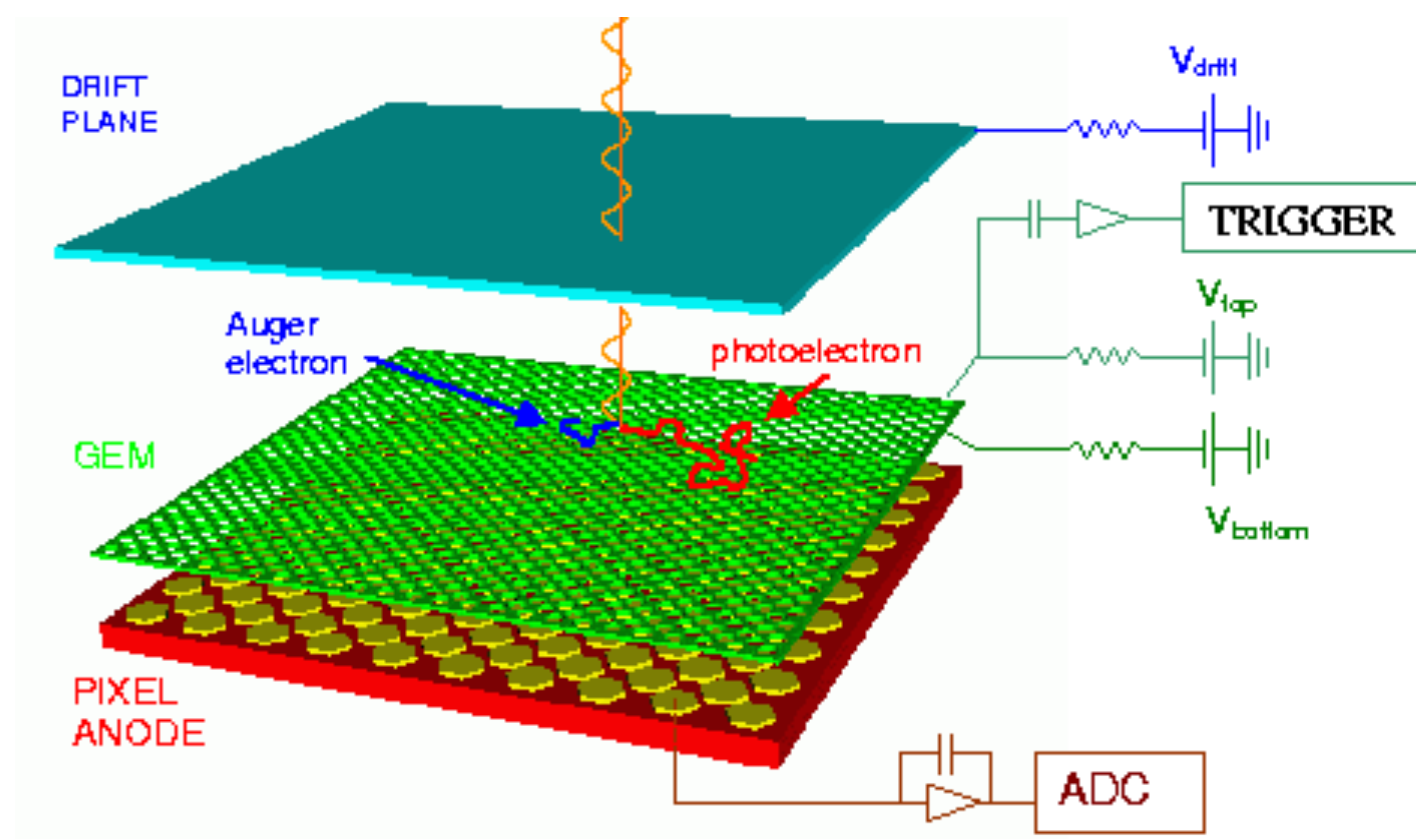


Imaging X-ray Polarization Explorer (IXPE)



IXPE Gas Pixel Detector

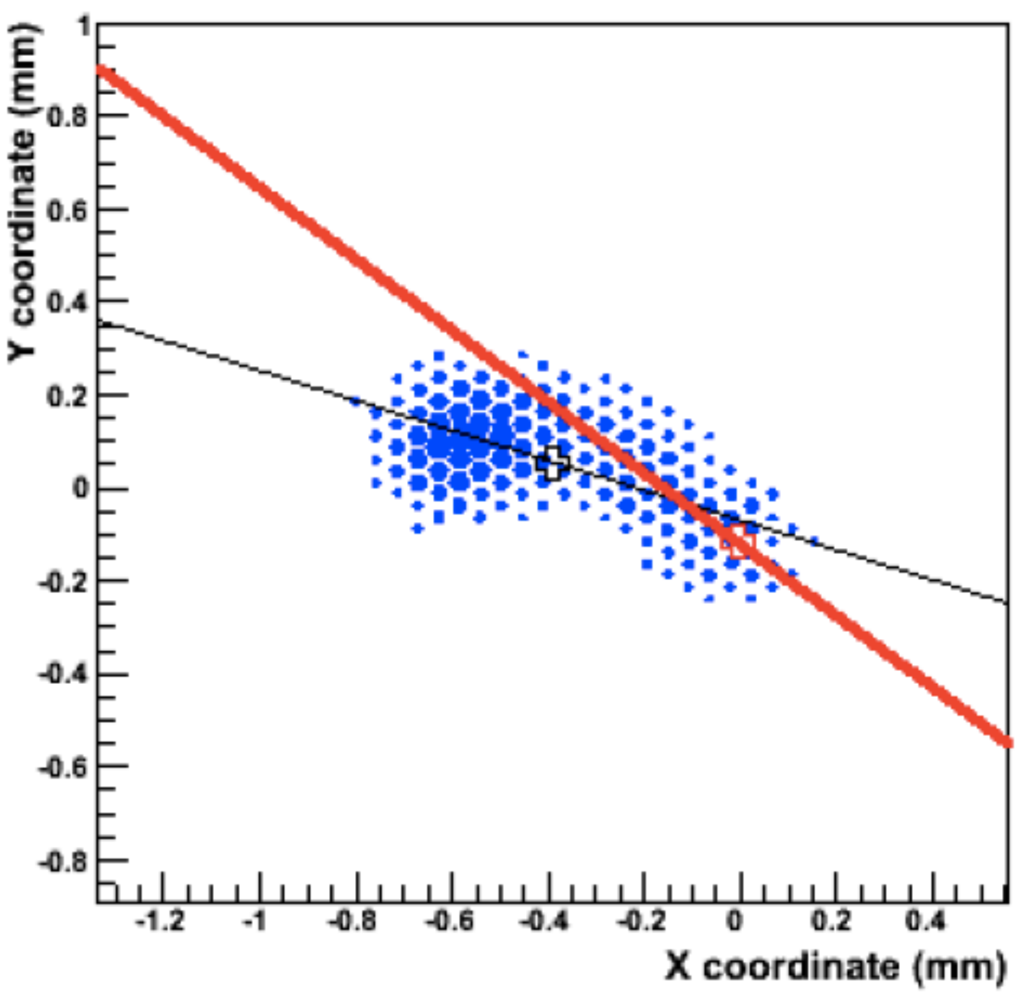


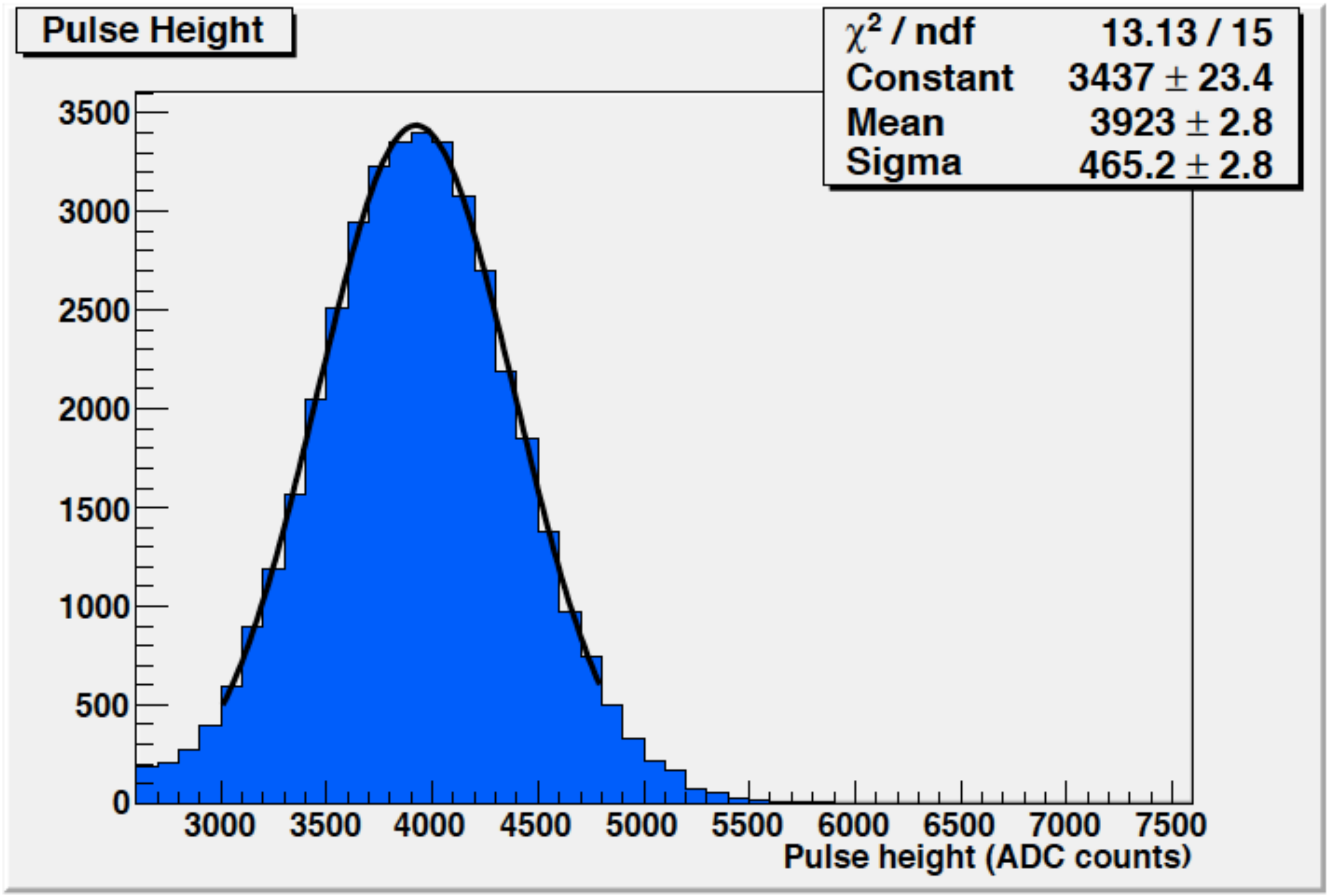


$$\frac{\partial\sigma}{\partial\Omega} = r_0^2 \frac{Z^5}{137^4}\left(\frac{mc^2}{h\nu}\right)^{7/2}\frac{4\sqrt{2}\sin^2(\theta)\cos^2(\varphi)}{(1-\beta\cos(\theta))^4}$$

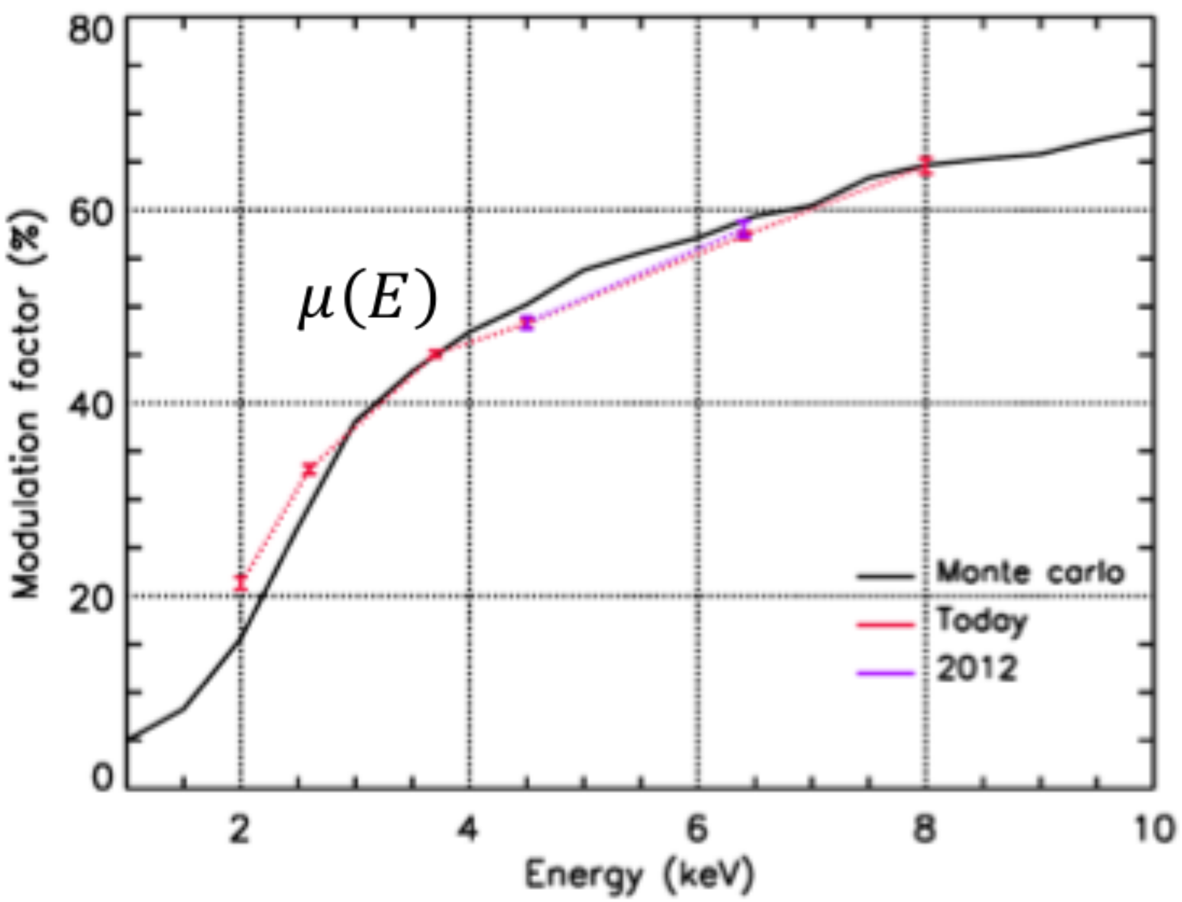
Event Results

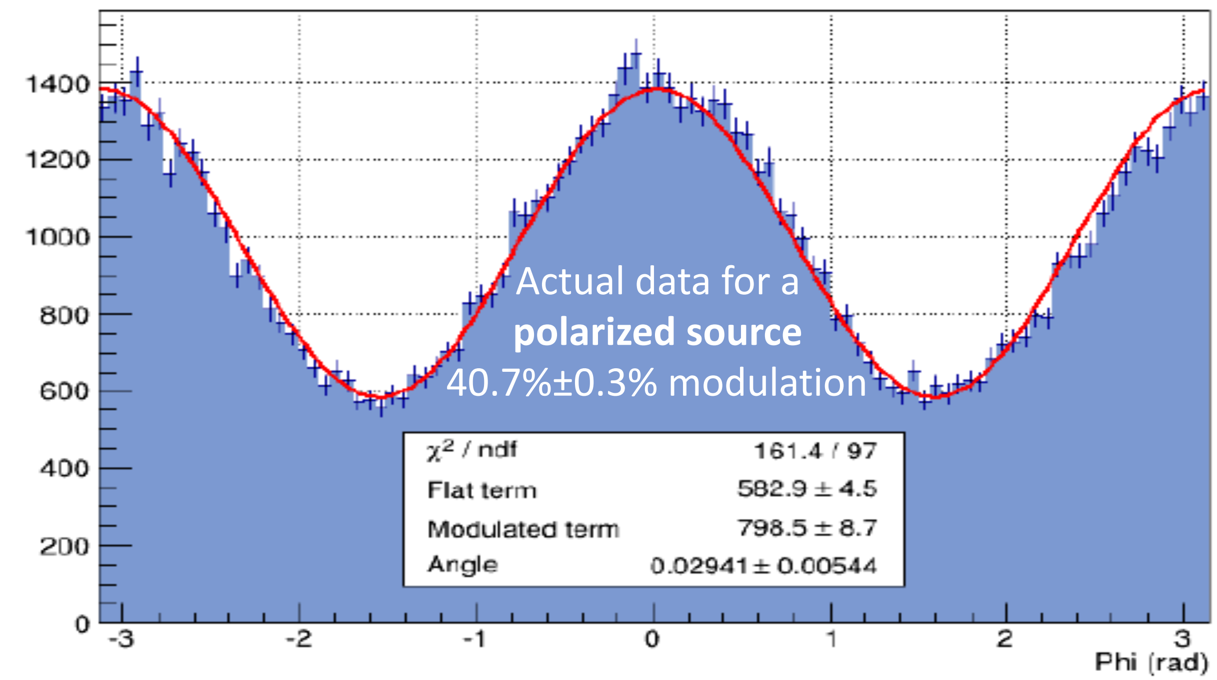
- Event time (to 10 μs), image, pulses measured
- Empirical method finds event origin, direction

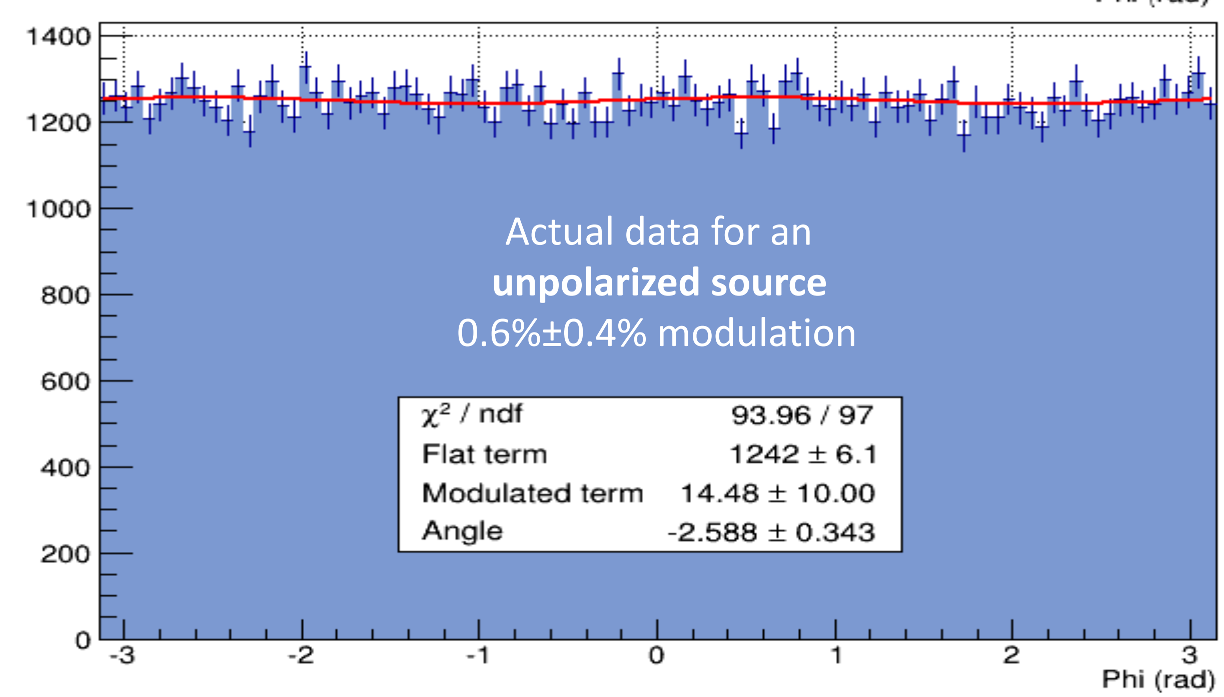




Polarization from modulation histogram and calibrated modulation factor







$$\mathrm{MDP}_{.99} = 3.035\sqrt{2}\frac{\sqrt{C_S + C_B}}{C_S\langle\mu\rangle}$$

The Standard Statistics

- Data: $X = \{x_i, y_i, dPH_i, t_i, \alpha_T(t_i), \delta_T(t_i)\}$, i = 1…N tracks
- Process X to $Y = \{\alpha_i, \delta_i, PH_i, t_i, \phi_i)$
- Use known distribution functions:
  - RMF: $R(PH \mid E) \sim G[\, gE, \sigma(E) \,]$
  - PSF: $F(\alpha, \delta, \mid \alpha_0, \delta_0)$
  - Polarization: $\lambda(E, \phi) = A(E)\, [\, I + \mu(E)\, Q \cos \phi + \mu(E)\, U \sin \phi \,]$
  - Generally, $I = f(E, t, \alpha_0, \delta_0)$, $Q = g(E, t, \alpha_0, \delta_0)$, $U = h(E, t, \alpha_0, \delta_0)$
  - Data are poisson: $Y \sim P(\, R^*F^*\lambda \,)$

Standard Analysis

$$\lambda_{ij} = [1 + \mu_j(q\cos 2\psi_i + u\sin 2\psi_i)] f_j A_j T \Delta E \Delta\psi$$

define $\alpha_j = f_j A_j T,\ \beta_j = \alpha_j \mu_j,\ s_i = \sin 2\psi_i,\ c_i = \cos 2\psi_i\ \sum_i s_i = \sum_i c_i = 0$

$$\chi^2 = \sum_i \sum_j \frac{(C_{ij} - \lambda_{ij})^2}{\sigma_{ij}^2} = \frac{1}{\sigma^2} \sum_i \sum_j (C_{ij} - [\alpha_j - q\beta_j c_i - u\beta_j s_i]\Delta E \Delta\psi)^2$$

$$\hat{\alpha}_j = \frac{\sum_i C_{ij}}{2\pi\Delta E} \quad \hat{f}_j = \frac{\sum_i C_{ij}}{2\pi T \Delta E A_j}$$

$$\sigma_{ij} = \sigma \quad \Delta' = \Delta\phi\Delta E$$

$$\hat{q} = \frac{\sum_i c_i \sum_j \beta_j C_{ij}}{\Delta' \sum_i c_i^2 \sum_j \beta_j^2} = \frac{n \sum_i c_i \sum_j \mu_j C_{ij} \sum_k C_{kj}}{\sum_i c_i^2 \sum_j \mu_j^2 (\sum_k C_{kj})^2}$$

$$\hat{u} = \frac{\sum_i s_i \sum_j \beta_j C_{ij}}{\Delta' \sum_i s_i^2 \sum_j \beta_j^2} = \frac{n \sum_i s_i \sum_j \mu_j C_{ij} \sum_k C_{kj}}{\sum_i s_i^2 \sum_j \mu_j^2 (\sum_k C_{kj})^2}$$

define $C_j = \sum_k C_{kj}$ and note that generally $\sum_i s_i^2 = \sum_i c_i^2 = n/2$, so

$$\hat{q} = \frac{2\sum_i c_i \sum_j \mu_j C_j C_{ij}}{\sum_j \mu_j^2 C_j^2}$$

$$\hat{u} = \frac{2\sum_i s_i \sum_j \mu_j C_j C_{ij}}{\sum_j \mu_j^2 C_j^2}$$

With uncertainties, but qµ << 1, uµ << 1

$$\hat{\alpha}_j = \frac{1}{\Delta'} \frac{\sum_i C_{ij}/\sigma_{ij}^2}{\sum_i 1/\sigma_{ij}^2} \equiv \frac{w_j}{\Delta'}$$

$$\hat{q}_j = \frac{2\sum_i c_i C_{ij}}{\mu_j C_{.j}} \qquad \hat{q} = \sum_j w'_j \hat{q}_j \qquad w'_j = \frac{\mu_j^2 w_j^2 \sum_i c_i^2/\sigma_{ij}^2}{\sum_k \mu_k^2 w_k^2 \sum_i c_i^2/\sigma_{ik}^2}$$

$$\hat{u}_j = \frac{2\sum_i s_i C_{ij}}{\mu_j C_{.j}} \qquad \hat{u} = \sum_j w''_j \hat{u}_j \qquad w''_j = \frac{\mu_j^2 w_j^2 \sum_i s_i^2/\sigma_{ij}^2}{\sum_k \mu_k^2 w_k^2 \sum_i s_i^2/\sigma_{ik}^2}$$

Standard Analysis, Unbinned

$$\lambda(E,\psi) = [1 + \mu_E(q\cos 2\psi + u\sin 2\psi)] f_E A_E T dE d\psi$$

$$S = -2\ln L \;= -2\sum_i \ln\lambda(E_i,\psi_i) + 2\int f_E A_E dE \int_0^{2\pi} d\psi[1 + \mu(E)(q\cos 2\psi + u\sin 2\psi)]$$

$$= -2\sum_i \ln f_i - 2\sum_i \ln(1 + q\mu_i\cos 2\psi_i + u\mu_i \sin 2\psi_i) + 4\pi T\int f_E A_E dE + \text{constant}$$

define $c_i = \mu_i\cos 2\psi_i$ and $s_i = \mu_i \sin 2\psi_i$ $\quad w_i = (1 + \hat{q}c_i + \hat{u}s_i)^{-1}$

$$0 = \sum_i \frac{s_i}{1+\hat{q}c_i+\hat{u}s_i} = \sum_i w_i s_i$$

$$0 = \sum_i \frac{c_i}{1+\hat{q}c_i+\hat{u}s_i} = \sum_i w_i c_i$$

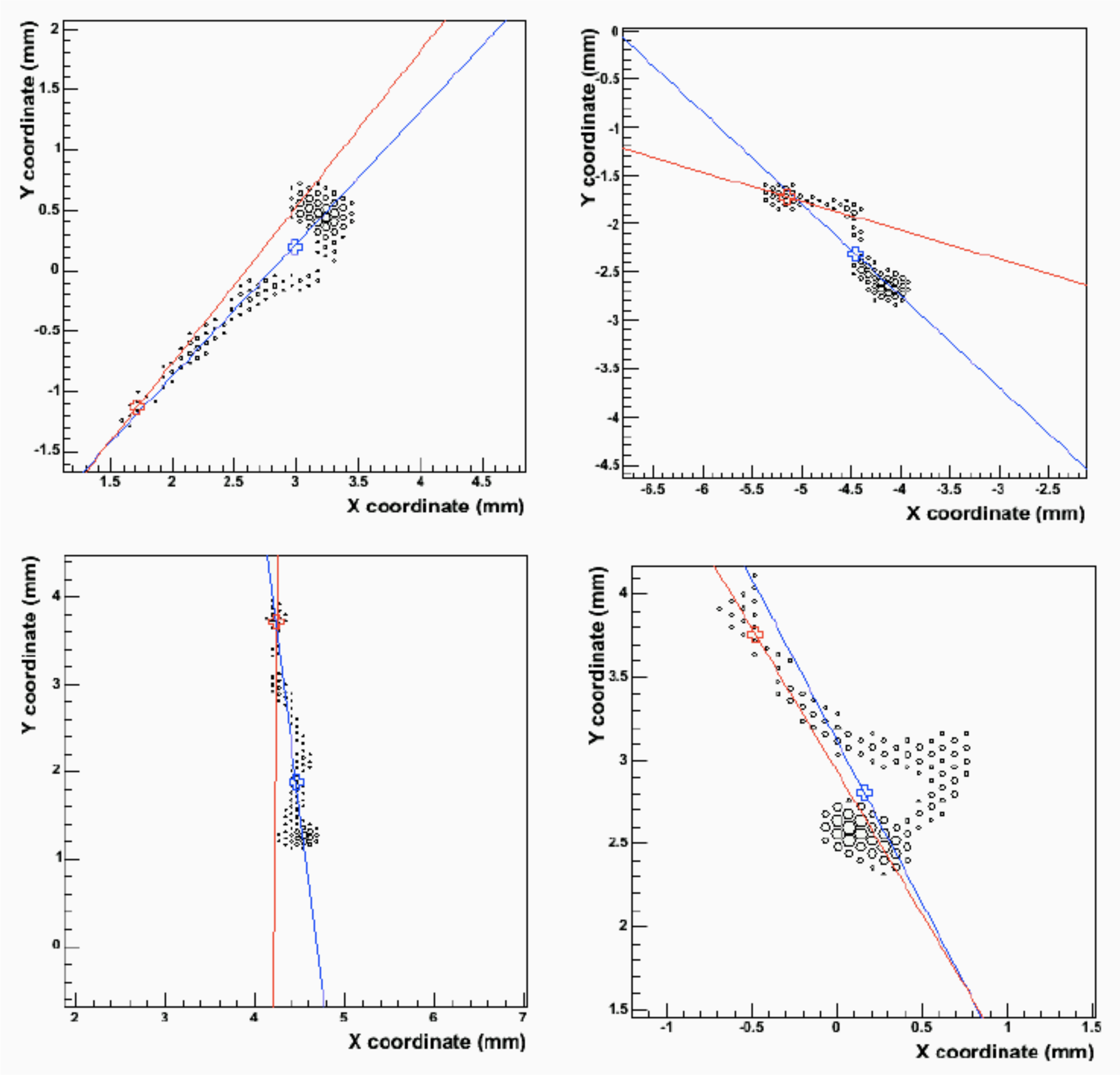
for $\hat{q} \ll 1$ and $\hat{u} \ll 1$

$$\sum s_i = \hat{u}\sum s_i^2 + \hat{q}\sum c_i s_i \qquad \hat{u} = \frac{\sum s_i \sum c_i^2 - \sum c_i \sum c_i s_i}{\sum s_i^2 \sum c_i^2 - (\sum c_i s_i)^2} \approx \frac{\sum s_i}{\sum s_i^2} \qquad \sigma_u^2 \approx \frac{2}{\frac{\partial^2 S}{\partial u^2}} \approx \frac{1}{\sum s_i^2}$$

$$\sum c_i = \hat{u}\sum c_i s_i + \hat{q}\sum c_i^2 \qquad \hat{q} = \frac{\sum c_i \sum s_i^2 - \sum s_i \sum c_i s_i}{\sum s_i^2 \sum c_i^2 - (\sum c_i s_i)^2} \approx \frac{\sum c_i}{\sum c_i^2} \qquad \sigma_q^2 \approx \frac{2}{\frac{\partial^2 S}{\partial q^2}} \approx \frac{1}{\sum c_i^2}$$

The Track Problem

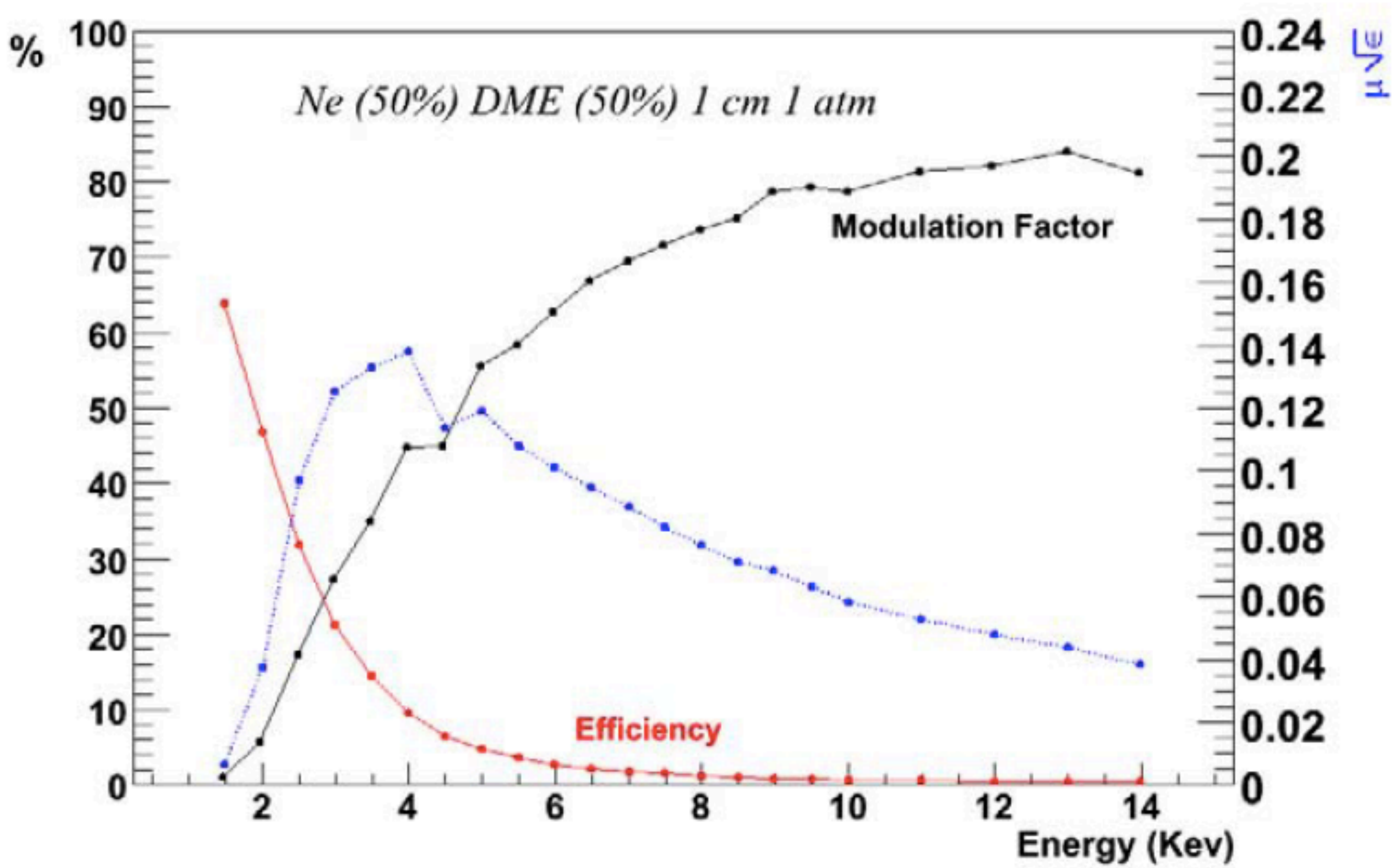
- Track measurement is empirical
  - Tracks have randomness
  - Bulk of PH is at (uninteresting) end of track
  - Low E tracks are short
  - Some events are not considered



- Tracks are only probabilistically related to X-ray polarization

- Tracks are measured independently

Track Algorithm Optimization



- MDP ~ 1/($\mu\ \varepsilon^{1/2}$)
- Algorithm has parameters that trade off μ and ε for best $\mu\ \varepsilon^{1/2}$

Simulated Data

```
XTENSION= 'BINTABLE'           / binary table extension
BITPIX  =                    8 / 8-bit bytes
NAXIS   =                    2 / 2-dimensional binary table
NAXIS1  =                   52 / width of table in bytes
NAXIS2  =                85995 / number of rows in table
PCOUNT  =             97967480 / size of special data area
GCOUNT  =                    1 / one data group (required keyword)
TFIELDS =                   12 / number of fields in each row
TTYPE1  = 'PAKTNUMB'           / label for field   1
TFORM1  = '1J      '           / data format of field: 4-byte INTEGER
TUNIT1  = '        '           / physical unit of field
TTYPE2  = 'TRG_ID  '           / label for field   2
TFORM2  = '1J      '           / data format of field: 4-byte INTEGER
TUNIT2  = '        '           / physical unit of field
TTYPE3  = 'SEC     '           / label for field   3
TFORM3  = '1K      '           / data format of field: 8-byte INTEGER
TUNIT3  = '        '           / physical unit of field
TTYPE4  = 'MICROSEC'           / label for field   4
TFORM4  = '1J      '           / data format of field: 4-byte INTEGER
TUNIT4  = '        '           / physical unit of field
TTYPE5  = 'TIME    '           / label for field   5
TFORM5  = '1D      '           / data format of field: 8-byte DOUBLE
TUNIT5  = '        '           / physical unit of field
TTYPE6  = 'MIN_COL '           / label for field   6
TFORM6  = '1I      '           / data format of field: 2-byte INTEGER
TUNIT6  = '        '           / physical unit of field
TTYPE7  = 'MAX_COL '           / label for field   7
TFORM7  = '1I      '           / data format of field: 2-byte INTEGER
TUNIT7  = '        '           / physical unit of field
TTYPE8  = 'MIN_ROW '           / label for field   8
TFORM8  = '1I      '           / data format of field: 2-byte INTEGER
TUNIT8  = '        '           / physical unit of field
TTYPE9  = 'MAX_ROW '           / label for field   9
TFORM9  = '1I      '           / data format of field: 2-byte INTEGER
TUNIT9  = '        '           / physical unit of field
TTYPE10 = 'ROI_SIZE'           / label for field  10
TFORM10 = '1J      '           / data format of field: 4-byte INTEGER
TUNIT10 = '        '           / physical unit of field
TTYPE11 = 'ERR_SUM '           / label for field  11
TFORM11 = '1J      '           / data format of field: 4-byte INTEGER
TUNIT11 = '        '           / physical unit of field
TTYPE12 = 'PIX_PHAS'           / label for field  12
TFORM12 = 'PI(3256)'           / data format of field: variable length array
TUNIT12 = '        '           / physical unit of field
EXTNAME = 'EVENTS  '           / name of this binary table extension
END
```

Simulated Data

```
XTENSION= 'BINTABLE'           / binary table extension
BITPIX  =                    8 / 8-bit bytes
NAXIS   =                    2 / 2-dimensional binary table
NAXIS1  =                   52 / width of table in bytes
NAXIS2  =                85995 / number of rows in table
PCOUNT  =             97967480 / size of special data area
GCOUNT  =                    1 / one data group (required keyword)
TFIELDS =                   12 / number of fields in each row
TTYPE1  = 'PAKTNUMB'           / label for field   1
TFORM1  = '1J      '           / data format of field: 4-byte INT
TUNIT1  = '        '           / physical unit of field
TTYPE2  = 'TRG_ID  '           / label for field   2
TFORM2  = '1J      '           / data format of field: 4-byte INT
TUNIT2  = '        '           / physical unit of field
TTYPE3  = 'SEC     '           / label for field   3
TFORM3  = '1K      '           / data format of field: 8-byte INT
TUNIT3  = '        '           / physical unit of field
TTYPE4  = 'MICROSEC'           / label for field   4
TFORM4  = '1J      '           / data format of field: 4-byte INT
TUNIT4  = '        '           / physical unit of field
TTYPE5  = 'TIME    '           / label for field   5
TFORM5  = '1D      '           / data format of field: 8-byte DOU
TUNIT5  = '        '           / physical unit of field
TTYPE6  = 'MIN_COL '           / label for field   6
TFORM6  = '1I      '           / data format of field: 2-byte INT
TUNIT6  = '        '           / physical unit of field
TTYPE7  = 'MAX_COL '           / label for field   7
TFORM7  = '1I      '           / data format of field: 2-byte INT
TUNIT7  = '        '           / physical unit of field
TTYPE8  = 'MIN_ROW '           / label for field   8
TFORM8  = '1I      '           / data format of field: 2-byte INT
TUNIT8  = '        '           / physical unit of field
TTYPE9  = 'MAX_ROW '           / label for field   9
TFORM9  = '1I      '           / data format of field: 2-byte INT
TUNIT9  = '        '           / physical unit of field
TTYPE10 = 'ROI_SIZE'           / label for field  10
TFORM10 = '1J      '           / data format of field: 4-byte INT
TUNIT10 = '        '           / physical unit of field
TTYPE11 = 'ERR_SUM '           / label for field  11
TFORM11 = '1J      '           / data format of field: 4-byte INT
TUNIT11 = '        '           / physical unit of field
TTYPE12 = 'PIX_PHAS'           / label for field  12
TFORM12 = 'PI(3256)'           / data format of field: variable l
TUNIT12 = '        '           / physical unit of field
EXTNAME = 'EVENTS  '           / name of this binary table extens
END
```

```
NAXIS1  =                  133 / width of table in bytes
NAXIS2  =                85995 / number of rows in table
PCOUNT  =            304599288 / size of special data area
GCOUNT  =                    1 / one data group (required keyword)
TFIELDS =                   18 / number of fields in each row
TTYPE1  = 'ENERGY  '           / label for field   1
TFORM1  = '1D      '           / data format of field: 8-byte DOUBLE
TUNIT1  = 'keV     '           / physical unit of field
TTYPE2  = 'ABS_X   '           / label for field   2
TFORM2  = '1D      '           / data format of field: 8-byte DOUBLE
TUNIT2  = 'mm      '           / physical unit of field
TTYPE3  = 'ABS_Y   '           / label for field   3
TFORM3  = '1D      '           / data format of field: 8-byte DOUBLE
TUNIT3  = 'mm      '           / physical unit of field
TTYPE4  = 'ABS_Z   '           / label for field   4
TFORM4  = '1D      '           / data format of field: 8-byte DOUBLE
TUNIT4  = 'mm      '           / physical unit of field
TTYPE5  = 'ABS_ELE '           / label for field   5
TFORM5  = '1A      '           / data format of field: ASCII Character
TTYPE6  = 'PE_ENE  '           / label for field   6
TFORM6  = '1D      '           / data format of field: 8-byte DOUBLE
TUNIT6  = 'keV     '           / physical unit of field
TTYPE7  = 'PE_PHI  '           / label for field   7
TFORM7  = '1D      '           / data format of field: 8-byte DOUBLE
TUNIT7  = 'rad     '           / physical unit of field
TTYPE8  = 'PE_THET '           / label for field   8
TFORM8  = '1D      '           / data format of field: 8-byte DOUBLE
TUNIT8  = 'rad     '           / physical unit of field
TTYPE9  = 'AUG_ENE '           / label for field   9
TFORM9  = '1D      '           / data format of field: 8-byte DOUBLE
TUNIT9  = 'keV     '           / physical unit of field
TTYPE10 = 'AUG_PHI '           / label for field  10
TFORM10 = '1D      '           / data format of field: 8-byte DOUBLE
TUNIT10 = 'rad     '           / physical unit of field
TTYPE11 = 'AUG_THET'           / label for field  11
TFORM11 = '1D      '           / data format of field: 8-byte DOUBLE
TUNIT11 = 'rad     '           / physical unit of field
TTYPE12 = 'TRK_LEN '           / label for field  12
TFORM12 = '1D      '           / data format of field: 8-byte DOUBLE
TUNIT12 = 'mm      '           / physical unit of field
TTYPE13 = 'RANGE   '           / label for field  13
TFORM13 = '1D      '           / data format of field: 8-byte DOUBLE
TUNIT13 = 'mm      '           / physical unit of field
TTYPE14 = 'NUM_PAIR'           / label for field  14
TFORM14 = '1J      '           / data format of field: 4-byte INTEGER
TTYPE15 = 'ION_POSX'           / label for field  15
TFORM15 = 'PD(180) '           / data format of field: variable length array
TUNIT15 = 'mm      '           / physical unit of field
```

Simulated Data

```
XTENSION= 'BINTABLE'           / binary table extension
BITPIX  =                    8 / 8-bit bytes
NAXIS   =                    2 / 2-dimensional binary table
NAXIS1  =                   52 / width of table in bytes
NAXIS2  =                85995 / number of rows in table
PCOUNT  =             97967480 / size of special data area
GCOUNT  =                    1 / one data group (required keyword)
TFIELDS =                   12 / number of fields in each row
TTYPE1  = 'PAKTNUMB'           / label for field   1
TFORM1  = '1J      '           / data format of field: 4-byte INT
TUNIT1  = '        '           / physic
TTYPE2  = 'TRG_ID  '           / label
TFORM2  = '1J      '           / data f
TUNIT2  = '        '           / physic
TTYPE3  = 'SEC     '           / label
TFORM3  = '1K      '           / data f
TUNIT3  = '        '           / physic
TTYPE4  = 'MICROSEC'           / label
TFORM4  = '1J      '           / data f
TUNIT4  = '        '           / physic
TTYPE5  = 'TIME    '           / label
TFORM5  = '1D      '           / data f
TUNIT5  = '        '           / physic
TTYPE6  = 'MIN_COL '           / label
TFORM6  = '1I      '           / data f
TUNIT6  = '        '           / physic
TTYPE7  = 'MAX_COL '           / label
TFORM7  = '1I      '           / data f
TUNIT7  = '        '           / physic
TTYPE8  = 'MIN_ROW '           / label
TFORM8  = '1I      '           / data f
TUNIT8  = '        '           / physic
TTYPE9  = 'MAX_ROW '           / label
TFORM9  = '1I      '           / data f
TUNIT9  = '        '           / physic
TTYPE10 = 'ROI_SIZE'           / label
TFORM10 = '1J      '           / data f
TUNIT10 = '        '           / physic
TTYPE11 = 'ERR_SUM '           / label
TFORM11 = '1J      '           / data f
TUNIT11 = '        '           / physic
TTYPE12 = 'PIX_PHAS'           / label
TFORM12 = 'PI(3256)'           / data f
TUNIT12 = '        '           / physic
EXTNAME = 'EVENTS  '           / name o
END
```

```
NAXIS1  =                  133 / width of table in bytes
NAXIS2  =                85995 / number of rows in table
PCOUNT  =            304599288 / size of special data area
GCOUNT  =                    1 / one data group (required keyword)
TFIELDS =                   18 / number of fields in each row
TTYPE1  = 'ENERGY  '           / label for field   1
TFORM1  = '1D      '           / data format of field: 8-byte DOUBLE
TUNIT1  = 'keV     '           / physical unit of field
...
TFORM15 = 'PD(180) '           / data format of field: variable length array
TUNIT15 = 'mm      '           / physical unit of field
```

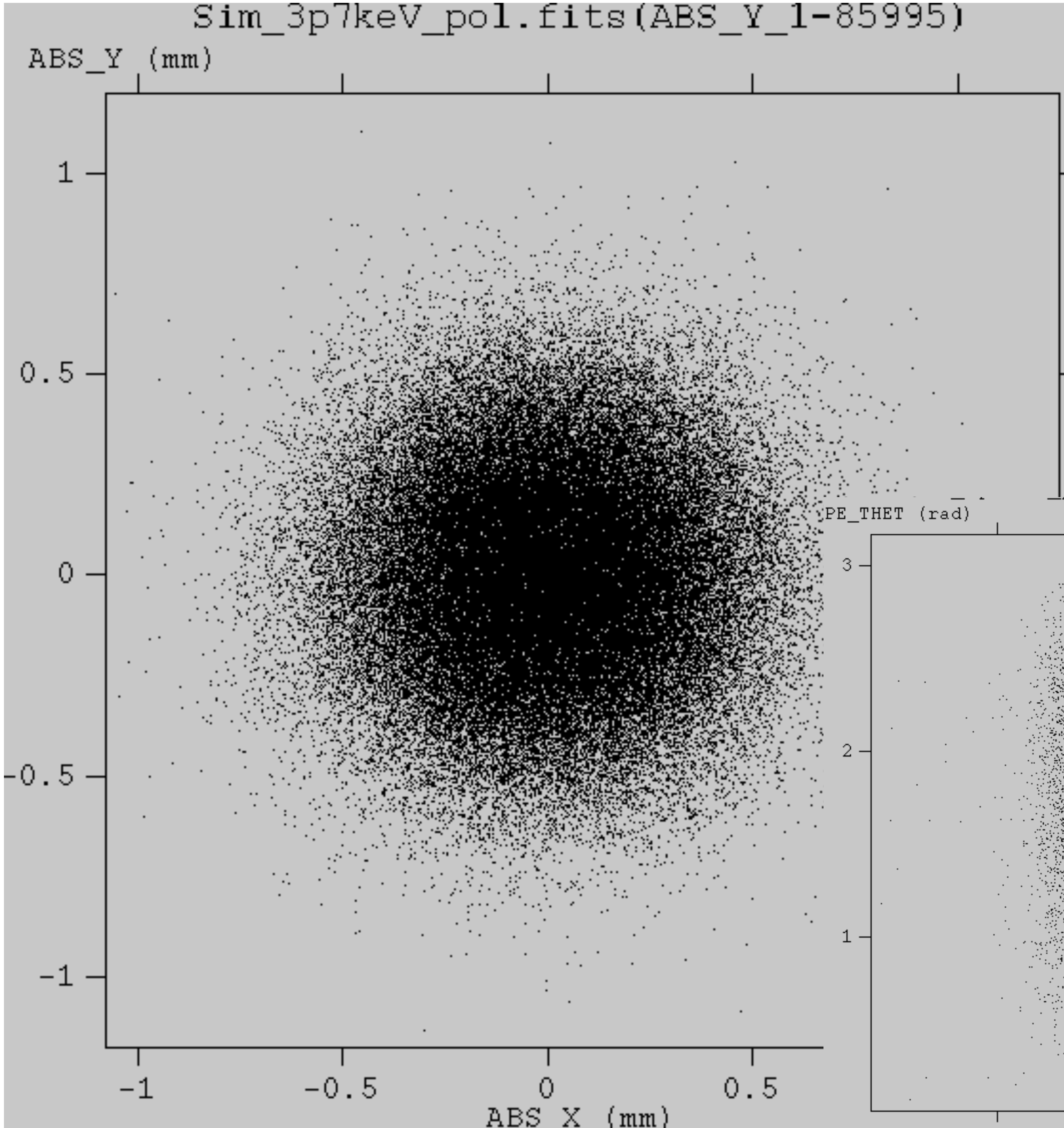
Select

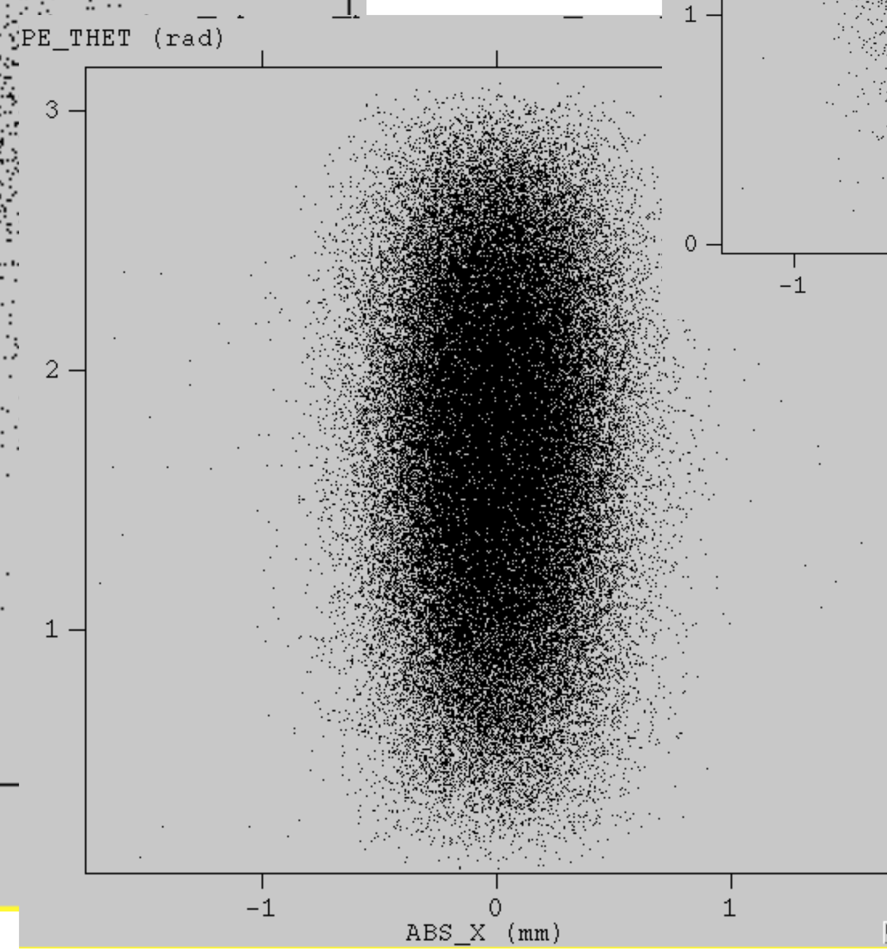
All

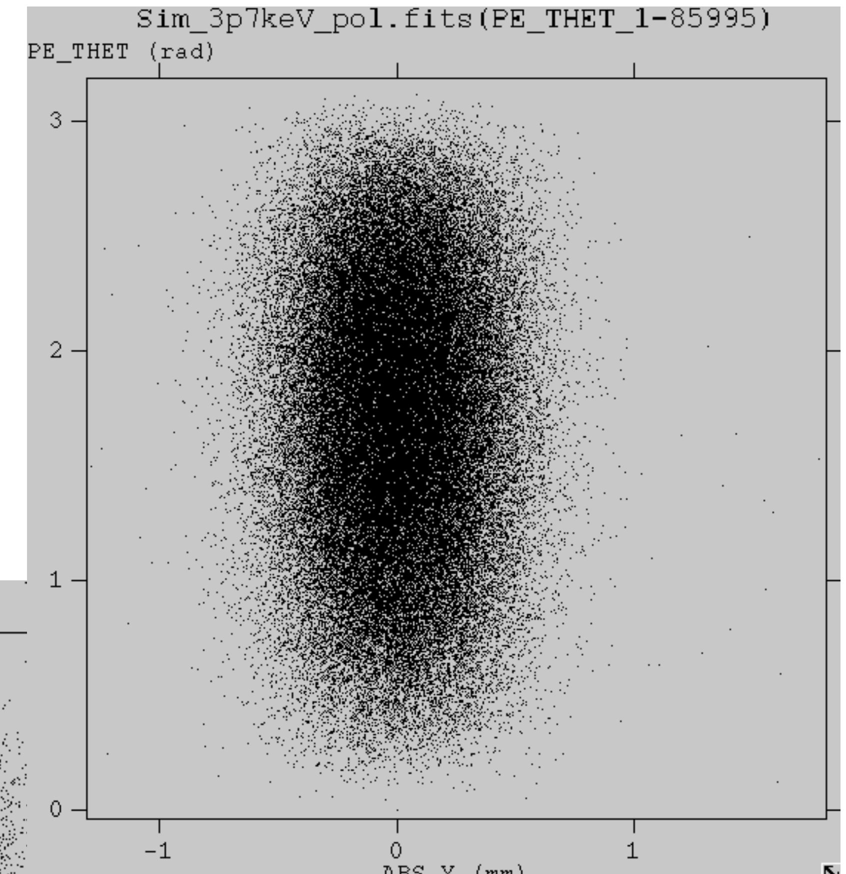
Invert

	1	2	3	4	5	6
1	2	4	5	1	4	8
2	4	5	3	2	5	3
3	2	5	3	1	0	4
4	1	1	3	5	2	2
5	0	3	2	0	5	1
6	5	2	1	0	2	0
7	7	6	2	3	2	4
8	3	2	9	2	3	6
9	1	2	4	1	4	1
10	1	3	1	1	1	1
11	6	2	0	1	1	1
12	5	5	2	7	1	2
13	3	0	4	3	2	2
14	8	11	2	5	4	4
15	1	3	5	2	2	0
16	3	3	3	3	1	5
17	7	3	4	8	1	4
18	2	3	2	1	1	5
19	3	2	4	2	1	4
20	1	8	5	2	0	1

Simulated Data







Calibration Data

- Known:
  - polarization angle
  - energy
  - source position
  - source is 100% polarized
- Detector data are ‘flight-like’
- Data are used to verify instrument model’s $\mu(E)$

Track Measurement via Machine Learning

HLM, Adam Trebach (MIT) and Michelle Ntampaka (CfA)

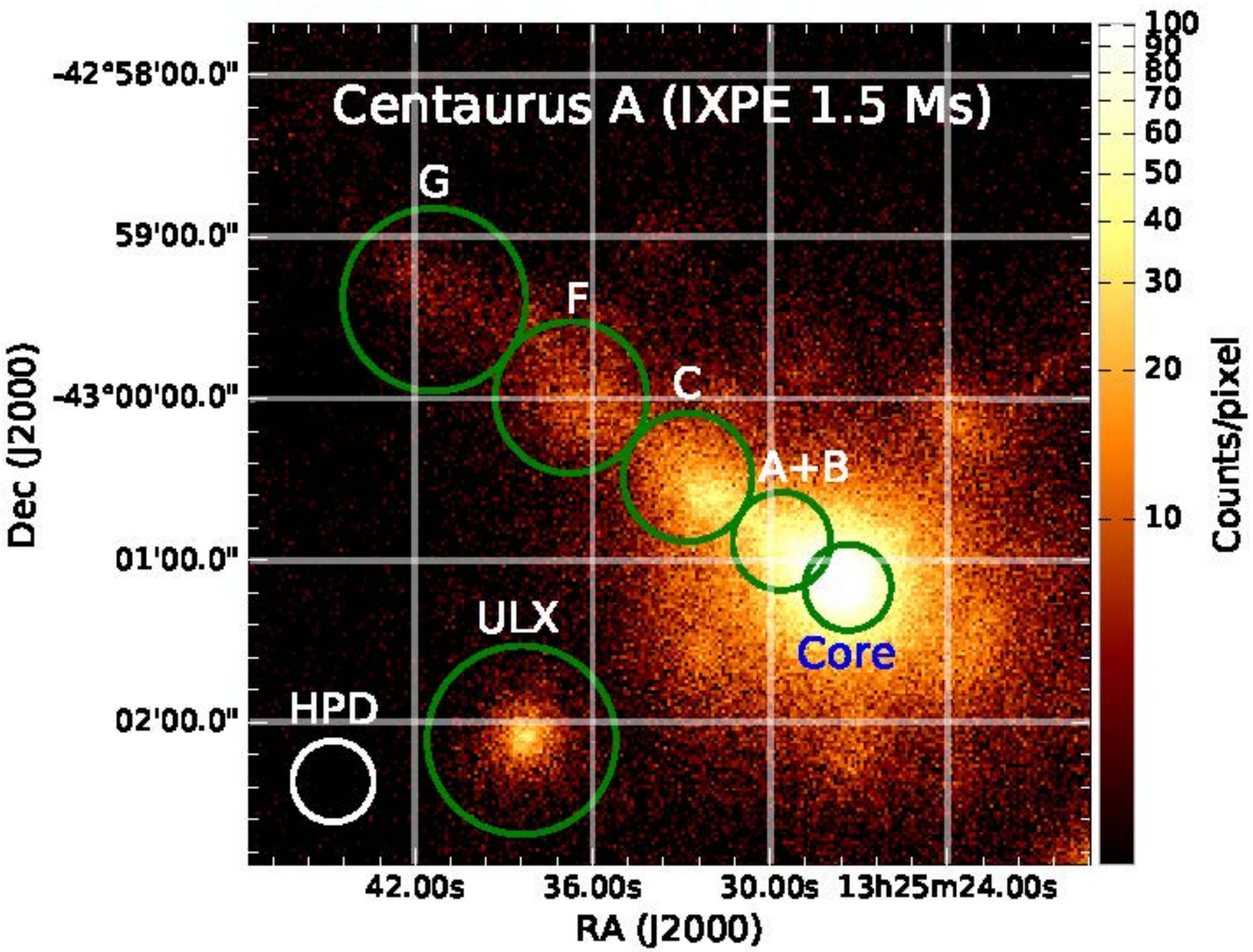
- Method 1 ('Tracking'): Learn track directions
  - Only trains with simulated data, needs physics of interaction
  - Event track is ~500 (x,y,PH) 3-tuples
  - Simulations have known photoelectron direction
  - Learns using ~10,000 events, apply to test sample of 1000 events
- Method 2 ('Holistic'): Learn polarization of event list
  - Trains on either simulated or calibration data
  - Training set is ~10,000 x 500 = 5 x 10^6 3-tuples
  - Polarization direction is known for training, applied to test data
  - Much faster than method 1

Model Fitting

- Traditional Method:
  - Bin I on (t, E, α, δ) into light curves, spectra, or images
  - Fit binned (or perhaps unbinned event list) using response functions
  - Handling complexity: time-dependent spectra, spatially varying spectra, etc: slice data in time or energy to make different spectra or images
- Problem: now add Q, U (or Π, ϕ)
  - Assume Π, ϕ are independent of E or t —> use traditional methods
  - Slice by E, α, δ (or t, α, δ) to get Π(E), $\phi(\alpha, \delta)$, etc.
- Alternative: Use priors based on Chandra (if unvarying) or joint observations
  - Requires Bayesian, multi-parameter modeling
  - Several scenarios are common

Constrain polarization properties of an imaged, bright AGN x-ray jet

- Centaurus A (Cen A = NGC 5128) central region
  - 1.5-Ms IXPE (simulated) observation of Cen A



Region	MDP_{99}
Core	**0.4%**
Jet	**10.9%**
Knot A+B	17.6%
Knot C	16.5%
Knot F	23.5%
Knot G	30.9%
ULX	**14.8%**

Model Testing

- Infeasible (?) using full track information
  - Tracks are not deterministically predictable
  - Derive distributions of general properties of tracks?
- Simplistic, easy: bin data, use χ^2
- Feasible, easy: unbinned K-S on ϕ, t, or E
- Challenging: Bayesian posterior
- Challenge: Simulation-based nearest neighbor test?