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Modeling our Milky Way

Astrostatistics | Big Data | Data Visualization

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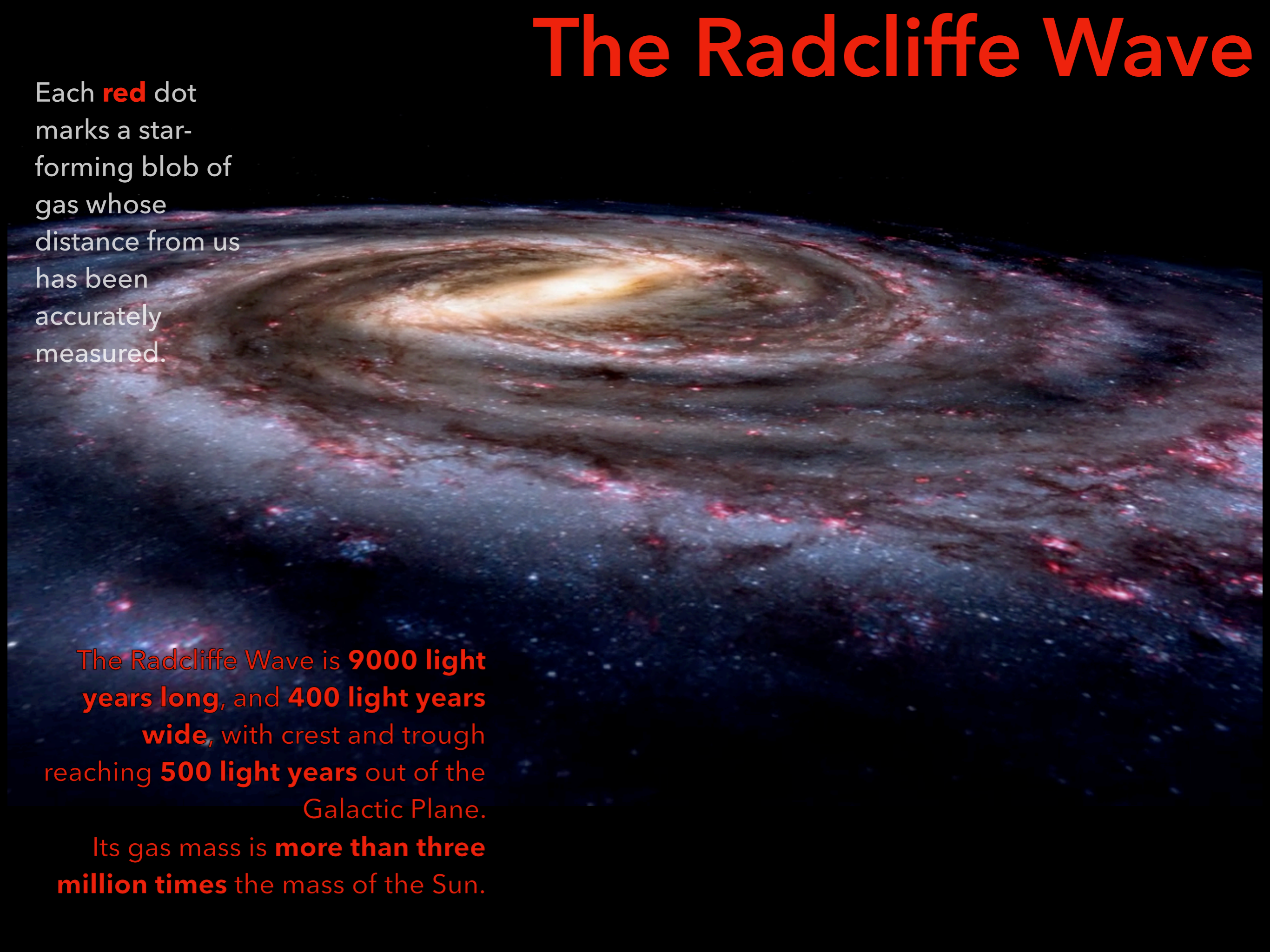
The last part, first.

The Radcliffe Wave

Each **red** dot marks a star-forming blob of gas whose distance from us has been accurately measured.

The Radcliffe Wave is **9000 light years long**, and **400 light years wide**, with crest and trough reaching **500 light years** out of the Galactic Plane.

Its gas mass is **more than three million times** the mass of the Sun.



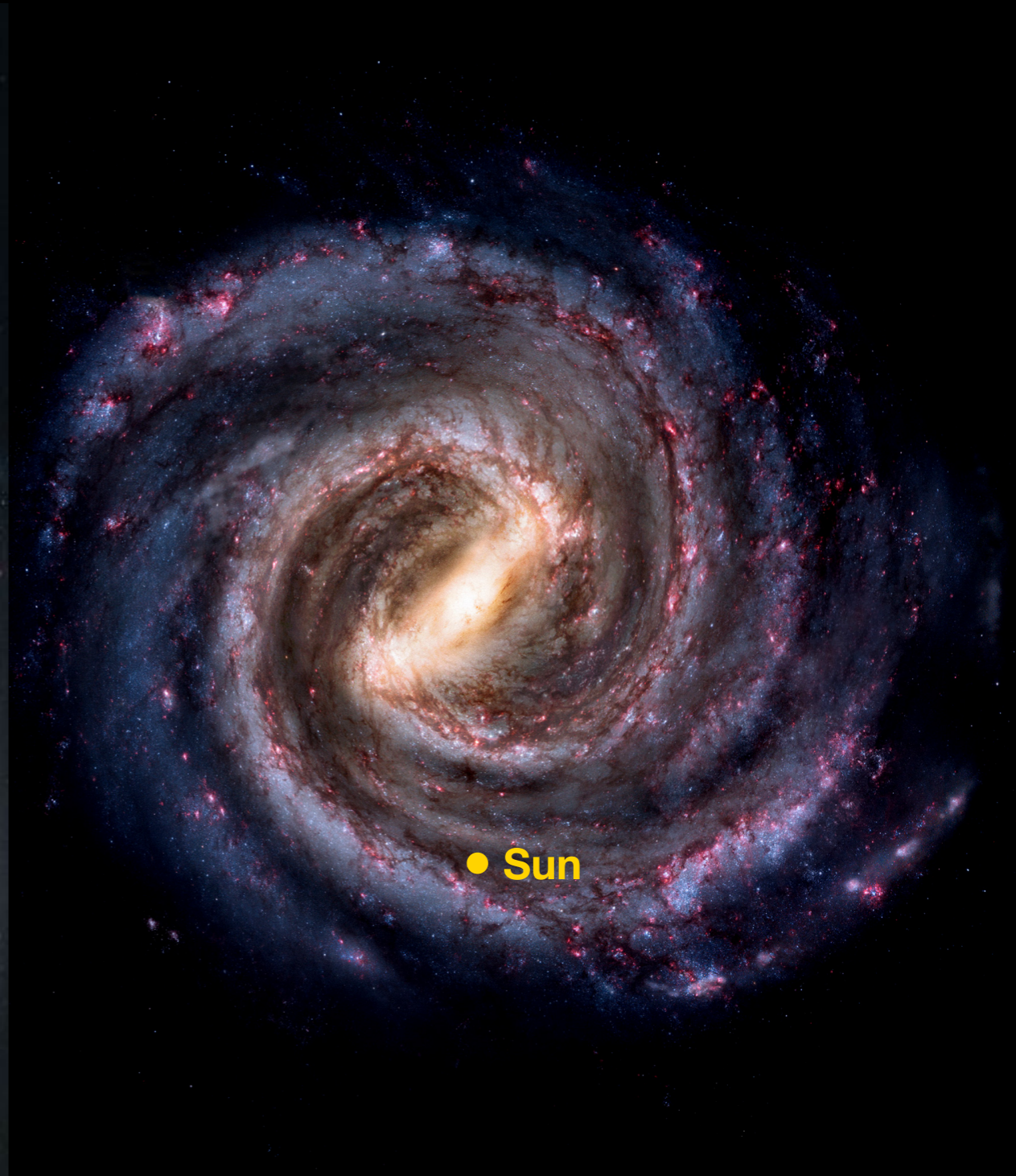


The Milky Way
(Artist's Conception)

Real & Fake



**Real Image of Actual
Spiral Galaxy**



**Cartoon Model of
our Milky Way**

Stellar Nursery



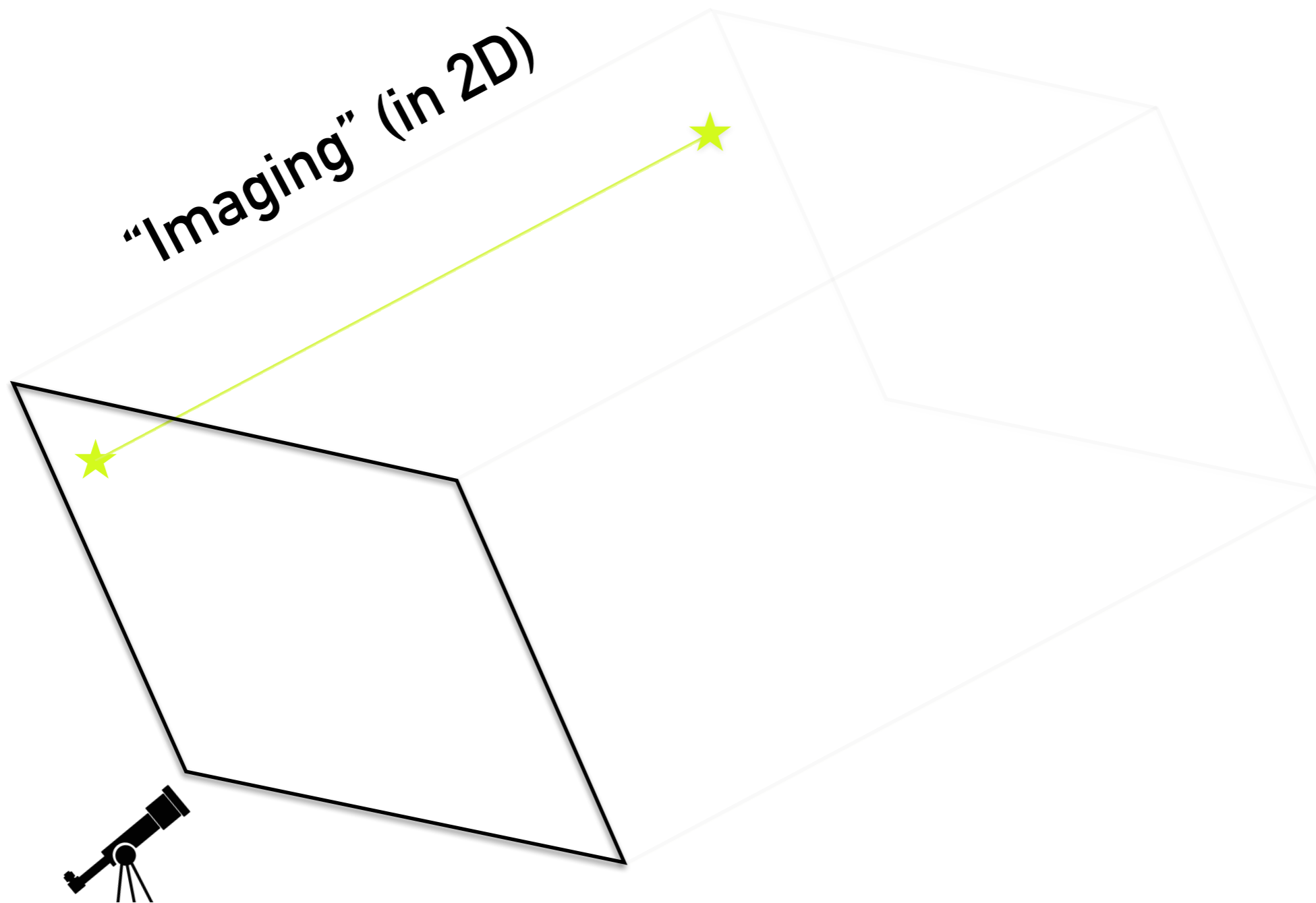
3D Cartoon



**Actual Image of the
Orion Nebula
(in 2D, on the sky)**

See Green+2014, 2015, 2018, 2019 for more details on stellar modeling

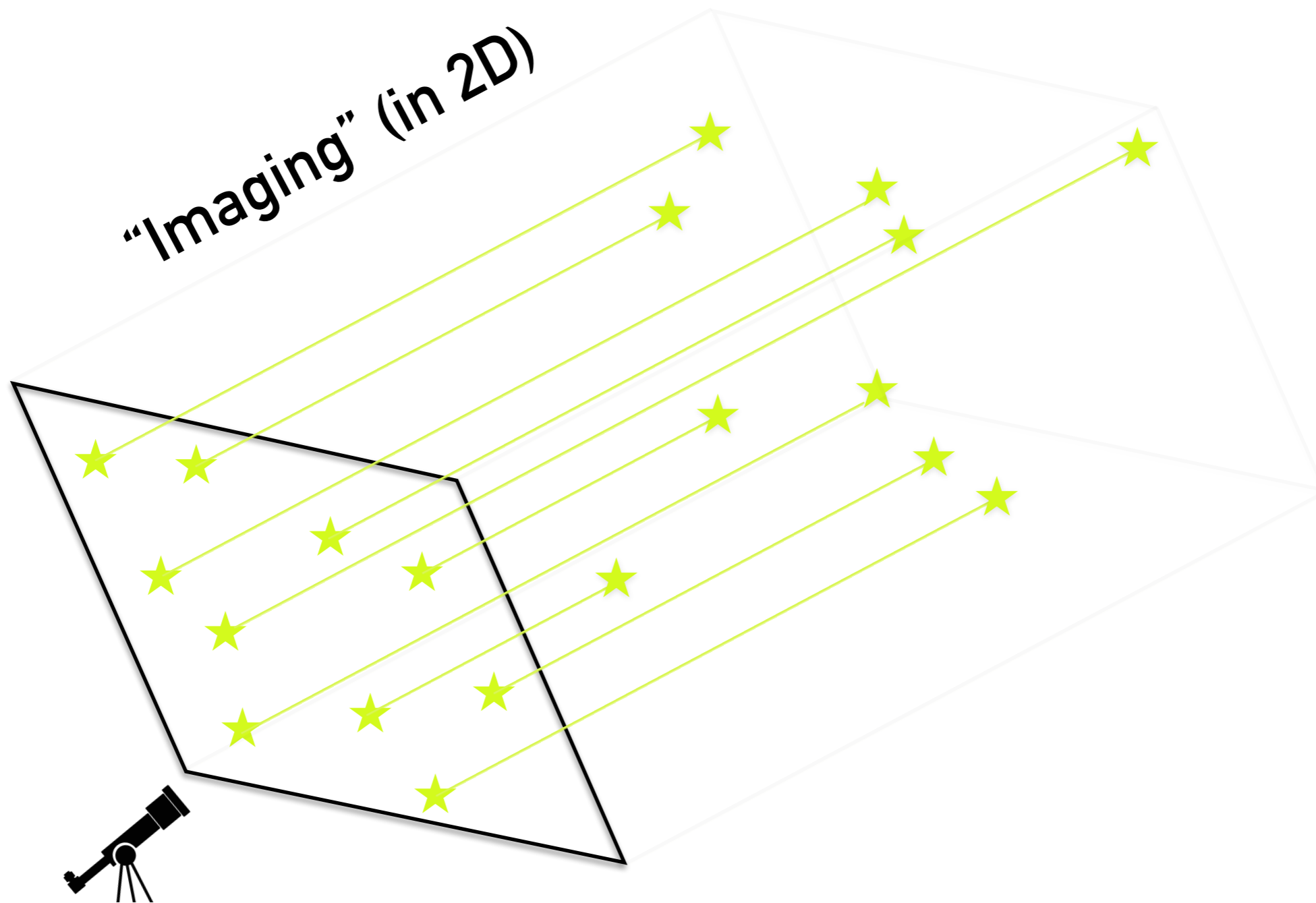
WARNING: schematic diagram, **NOT** to scale (credit A. Goodman, 2019)



gaia

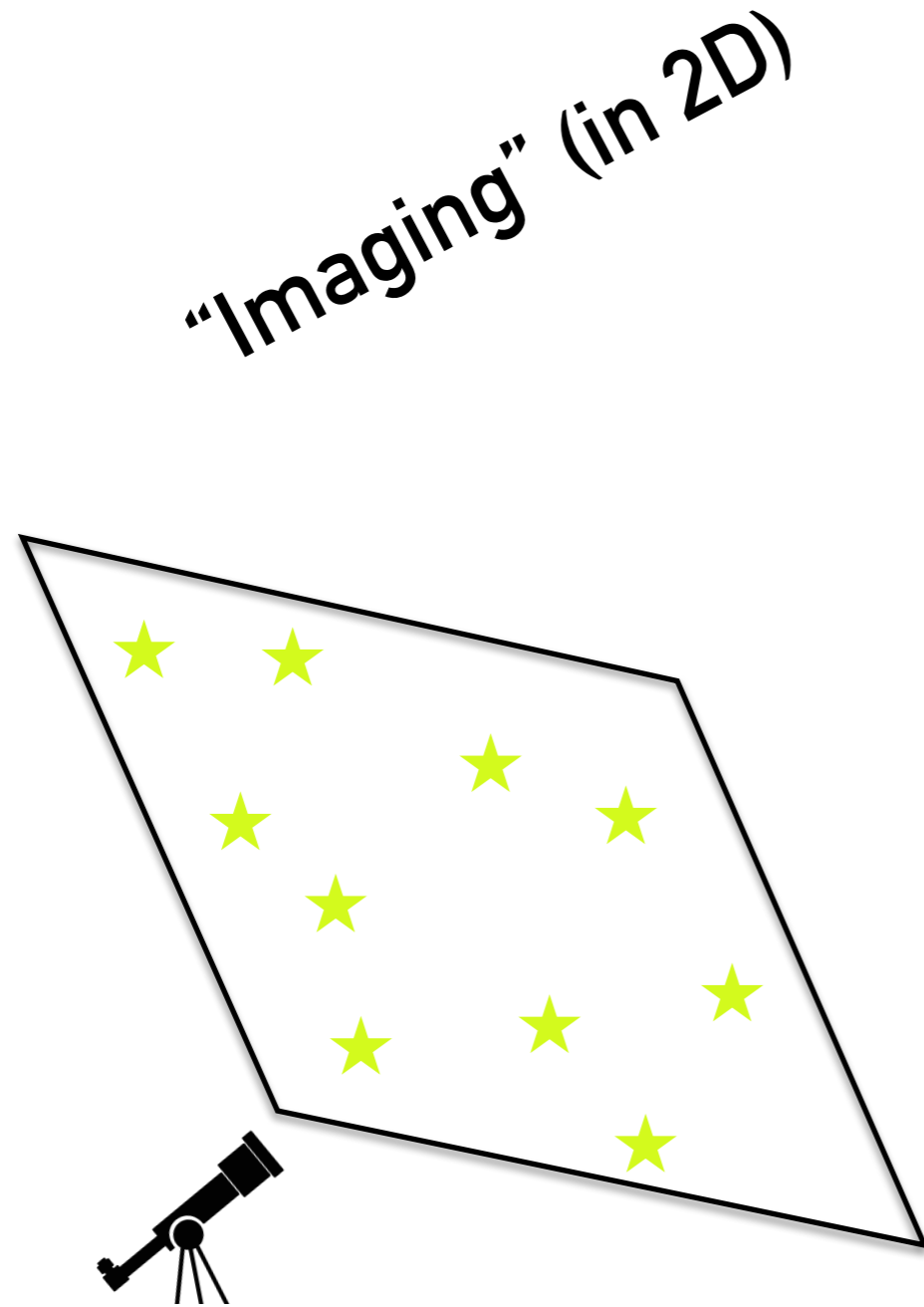
See Green+2014, 2015, 2018, 2019 for more details on stellar modeling

WARNING: schematic diagram, **NOT** to scale (credit A. Goodman, 2019)



See Green+2014, 2015, 2018, 2019 for more details on stellar modeling

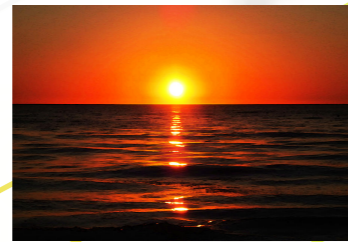
WARNING: schematic diagram, **NOT** to scale (credit A. Goodman, 2019)



+



Extinction & Reddening, from Color Imaging



Can infer cloud's distance from dust's effects on stars.



Methodology

[From star colors to a 3D map of (at least part of) the Galaxy]

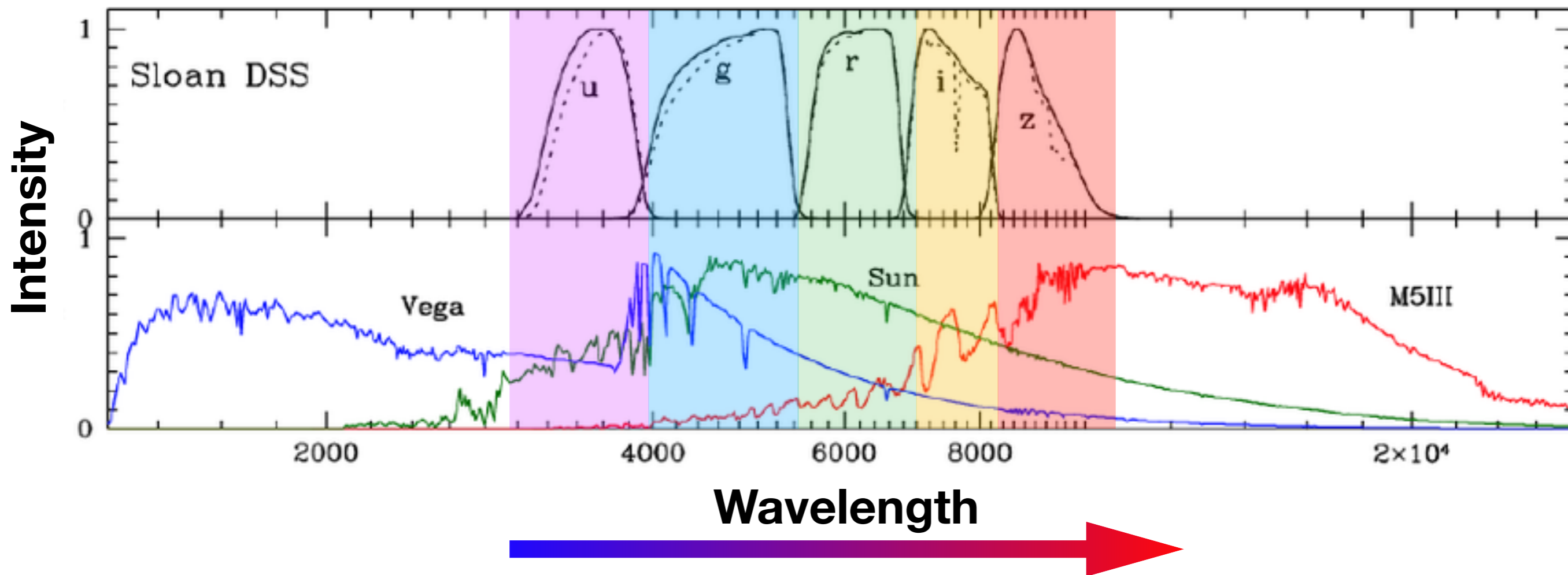
Part I.

Inferring Stellar Properties

How do we measure “star colors”? An introduction to photometry.

Photometry = set of “magnitudes” for each star:

$$\mathbf{m} = \{m_u, m_g, m_r, m_i, m_z, \dots\}$$

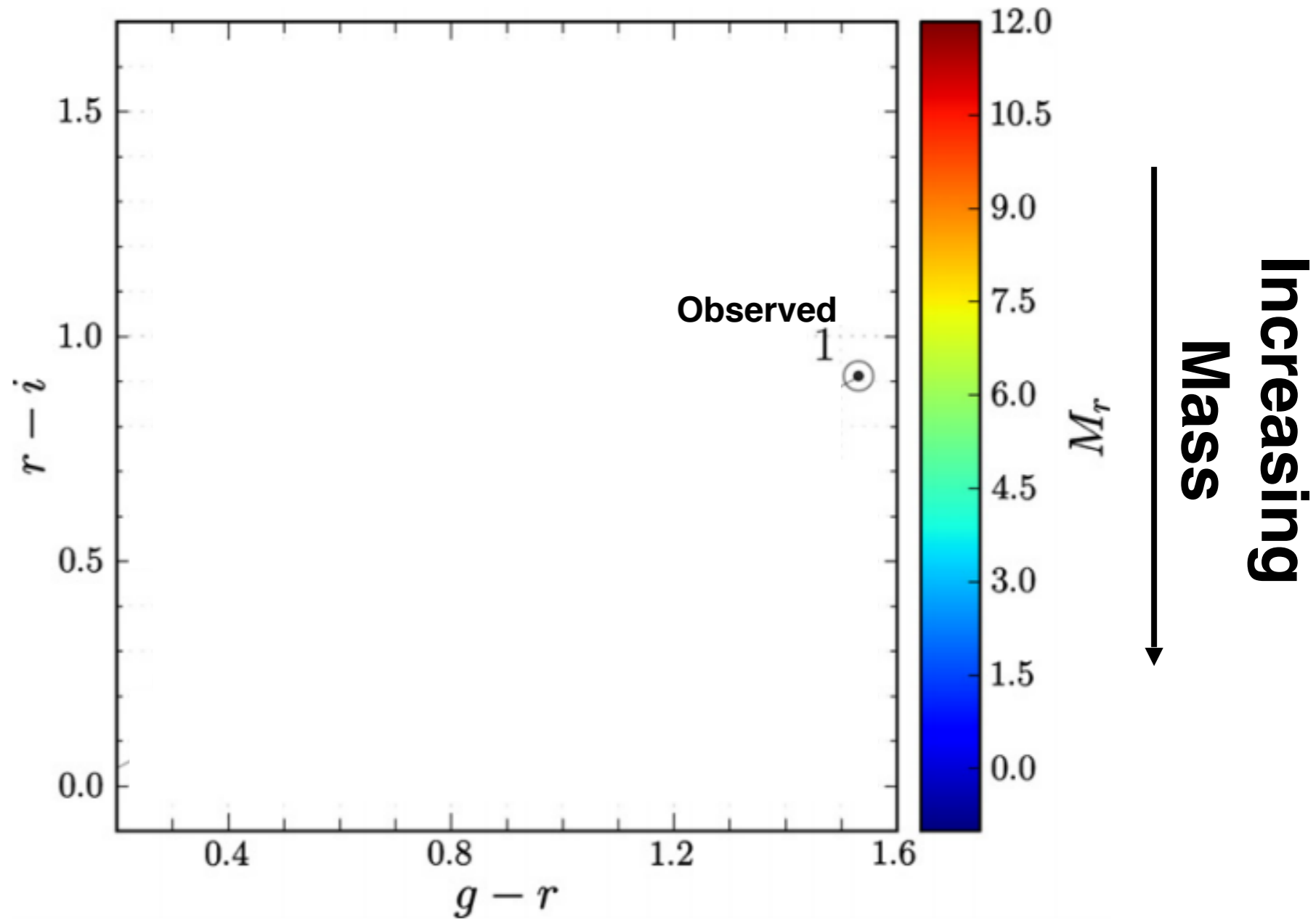


Photometry is available for **BILLIONS** of stars. Spectroscopy is available for millions.

Basic Idea

“Color-Color Space”

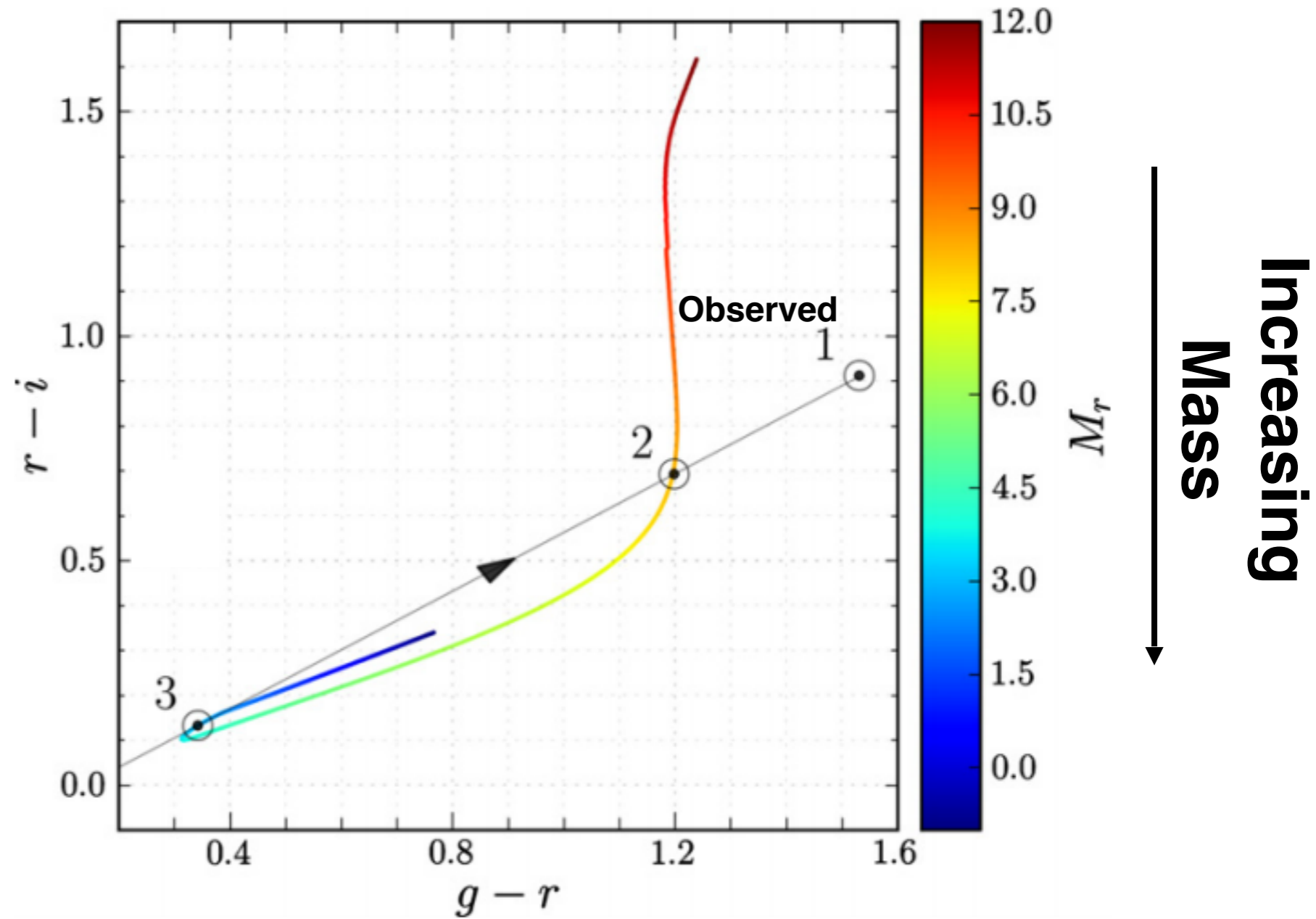
Green et al. (2014)



Basic Idea

“Color-Color Space”

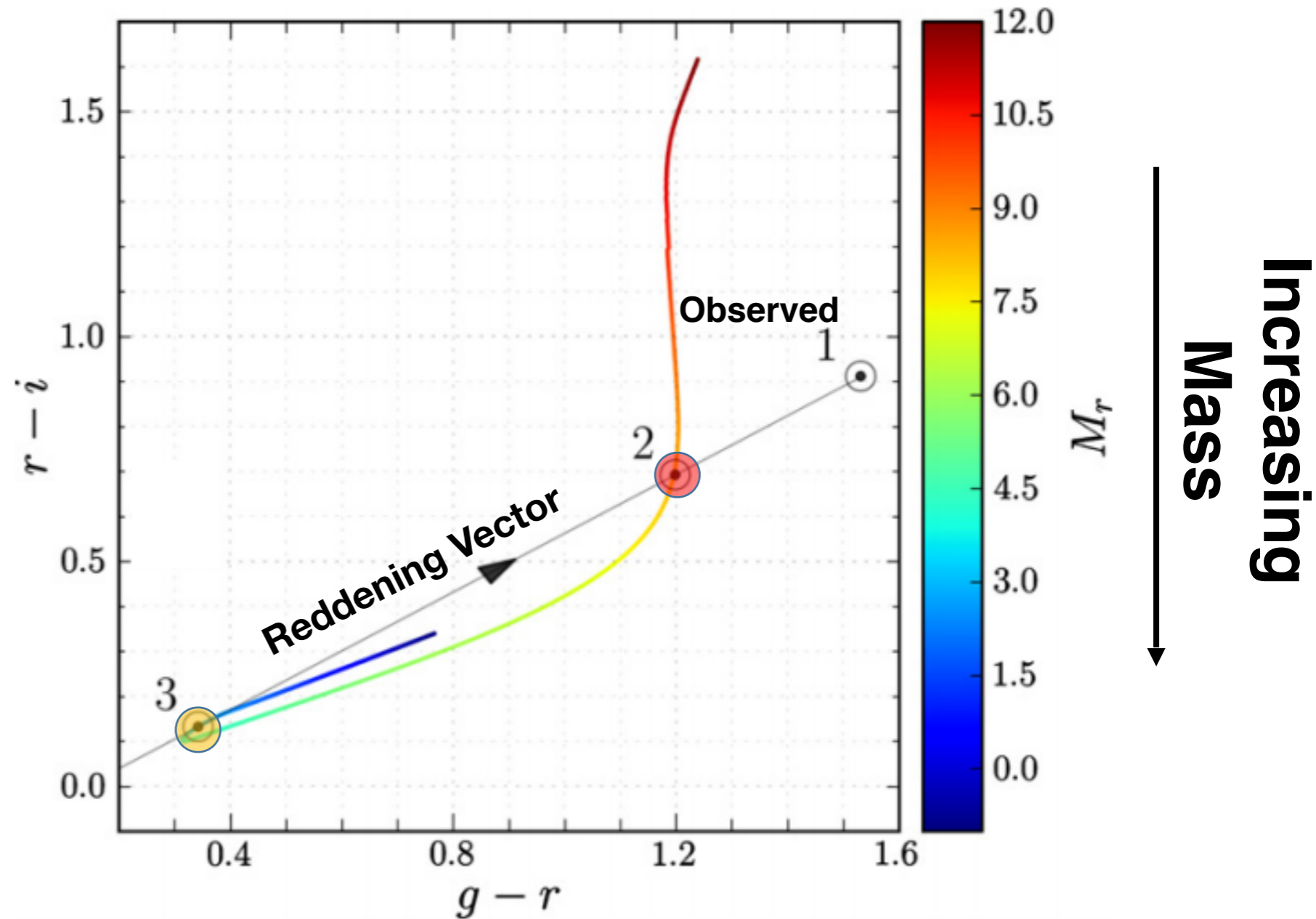
Green et al. (2014)



Basic Idea

“Color-Color Space”

Green et al. (2014)



- **Low-mass solution**

- Less reddening
- Closer

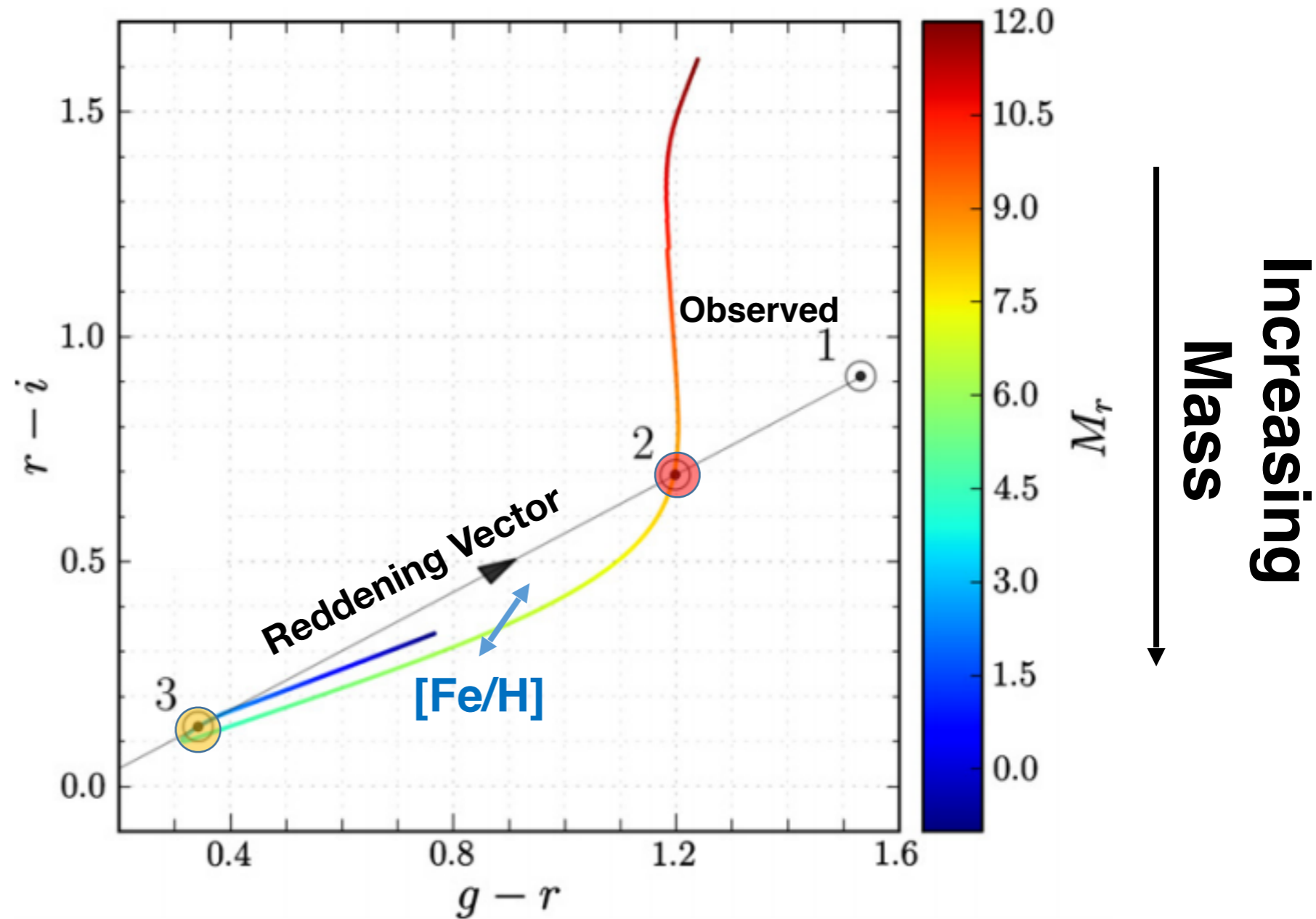
- **High-mass solution**

- More reddening
- Further

Basic Idea

“Color-Color Space”

Green et al. (2014)



● **Low-mass solution**

- Less reddening
- Closer

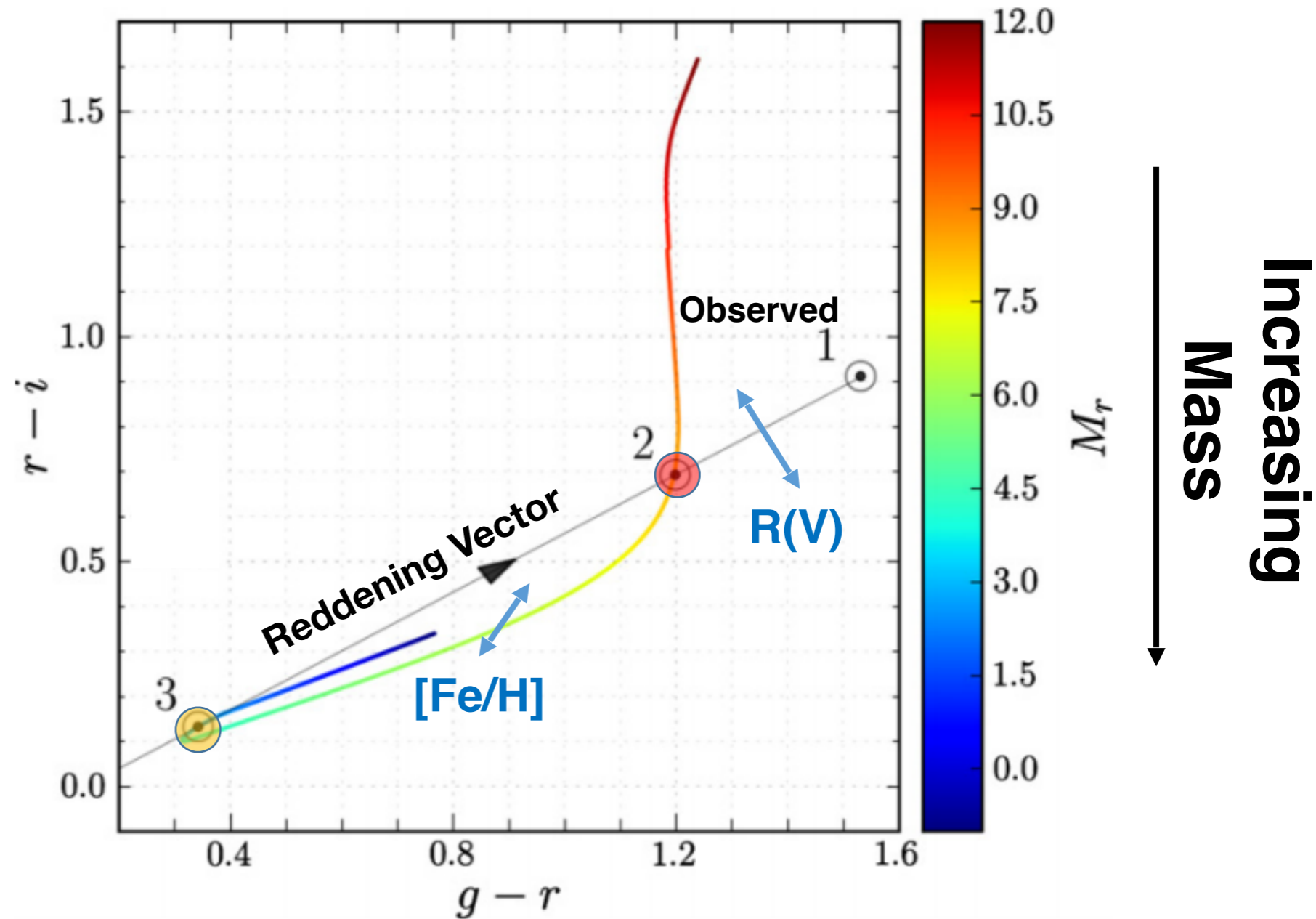
● **High-mass solution**

- More reddening
- Further

Basic Idea

“Color-Color Space”

Green et al. (2014)



- **Low-mass solution**

- Less reddening
- Closer

- **High-mass solution**

- More reddening
- Further

Stellar Inference

- We model each star as having (predicted) observed magnitudes \mathbf{m} .
- \mathbf{m} is a function of:
 1. “type” of star (M_r , $[\text{Fe}/\text{H}]$)
 2. reddening from dust (A_V , R_V)
 3. distance (μ)

$$\mathbf{m} = \mathbf{m}_{\text{int}}(M_r, [\text{Fe}/\text{H}]) + A_V(\mathbf{R} + R_V \mathbf{R}') + \mu$$

- Five-parameter model

$$\mathbf{m}(\boldsymbol{\theta}) \equiv \mathbf{m}(M_r, [\text{Fe}/\text{H}], A_V, R_V, \mu)$$

Stellar Inference

The posterior probability that observed magnitudes $\hat{\mathbf{m}}$ are consistent with our predicted photometry $\mathbf{m}(\boldsymbol{\theta})$ follows Bayes Rule:

Posterior

$$P(\boldsymbol{\theta} | \hat{\mathbf{m}}, \hat{\tau}) \propto \mathcal{L}(\hat{\mathbf{m}} | \boldsymbol{\theta}) \mathcal{L}(\hat{\tau} | \mu) \pi(\boldsymbol{\theta})$$

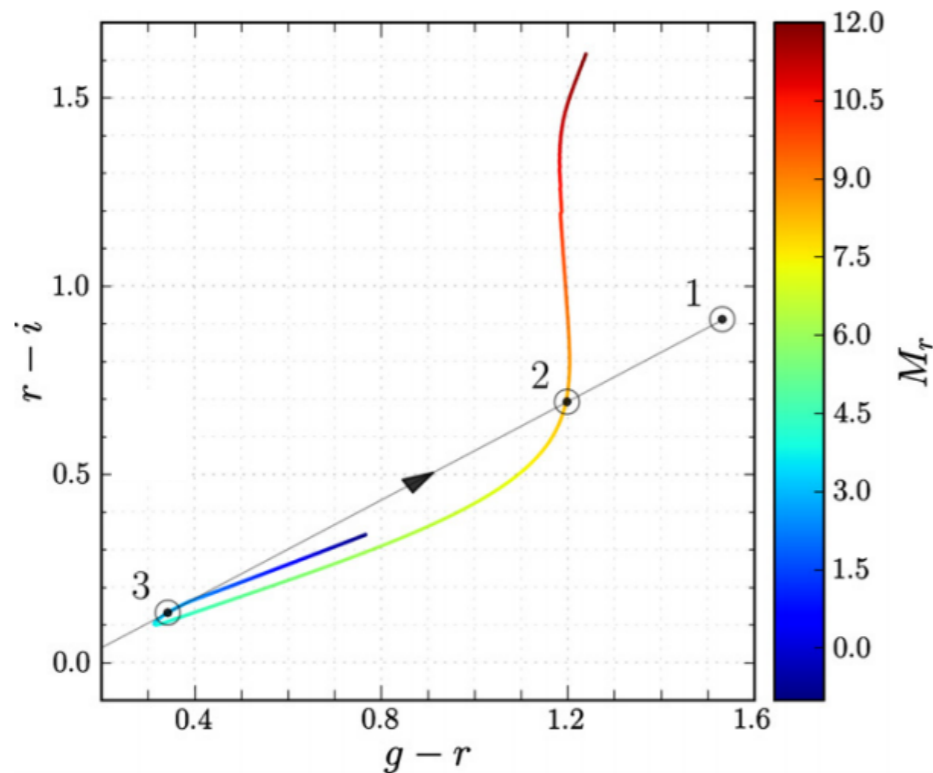
$\boldsymbol{\theta}$: stellar type, reddening,
distance

Stellar Inference

The posterior probability that observed magnitudes $\hat{\mathbf{m}}$ are consistent with our predicted photometry $\mathbf{m}(\boldsymbol{\theta})$ follows Bayes Rule:

Photometric
Likelihood

$$P(\boldsymbol{\theta} | \hat{\mathbf{m}}, \hat{\tau}) \propto \mathcal{L}(\hat{\mathbf{m}} | \boldsymbol{\theta}) \mathcal{L}(\hat{\tau} | \mu) \pi(\boldsymbol{\theta})$$



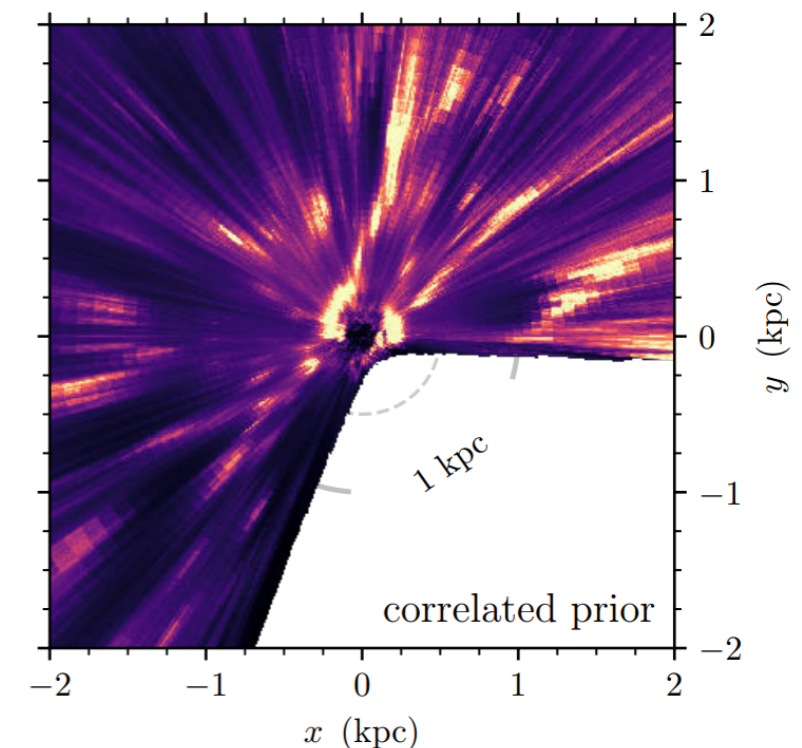
$$\mathcal{L}(\hat{\mathbf{m}} | \boldsymbol{\theta}) = \prod_b \frac{1}{\sqrt{2\pi}\sigma_b} \exp \left[-\frac{1}{2} \frac{(\mathbf{m}(\boldsymbol{\theta}) - \hat{\mathbf{m}})^2}{\sigma_b^2} \right]$$

Stellar Inference

The posterior probability that observed magnitudes $\hat{\mathbf{m}}$ are consistent with our predicted magnitudes $\mathbf{m}(\boldsymbol{\theta})$ follows Bayes Rule:

$$P(\boldsymbol{\theta} | \hat{\mathbf{m}}, \hat{\tau}) \propto \mathcal{L}(\hat{\mathbf{m}} | \boldsymbol{\theta}) \mathcal{L}(\hat{\tau} | \mu) \boxed{\pi(\boldsymbol{\theta})} \quad \text{Priors}$$

3-D Galactic model



Stellar Inference

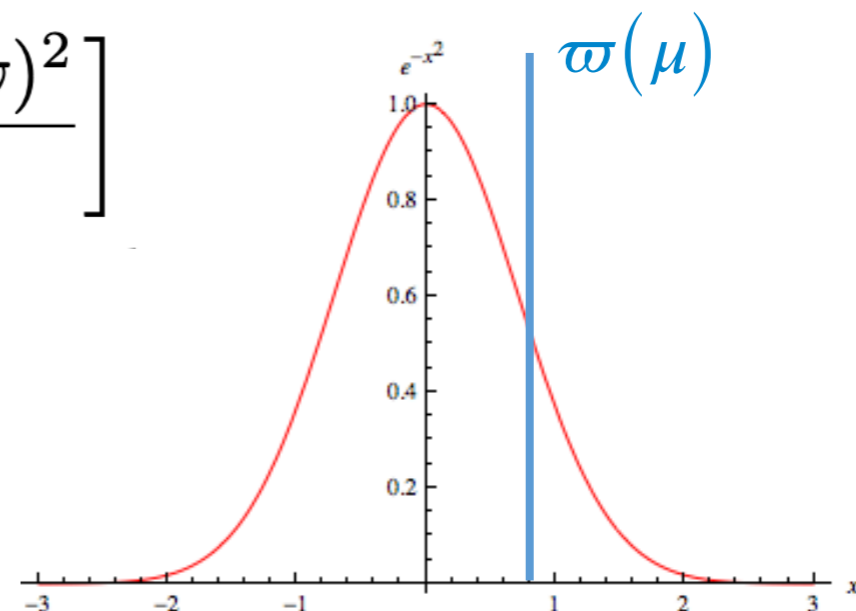
The posterior probability that observed magnitudes $\hat{\mathbf{m}}$ are consistent with our predicted photometry $\mathbf{m}(\boldsymbol{\theta})$ follows Bayes Rule:

**Parallax
Likelihood**

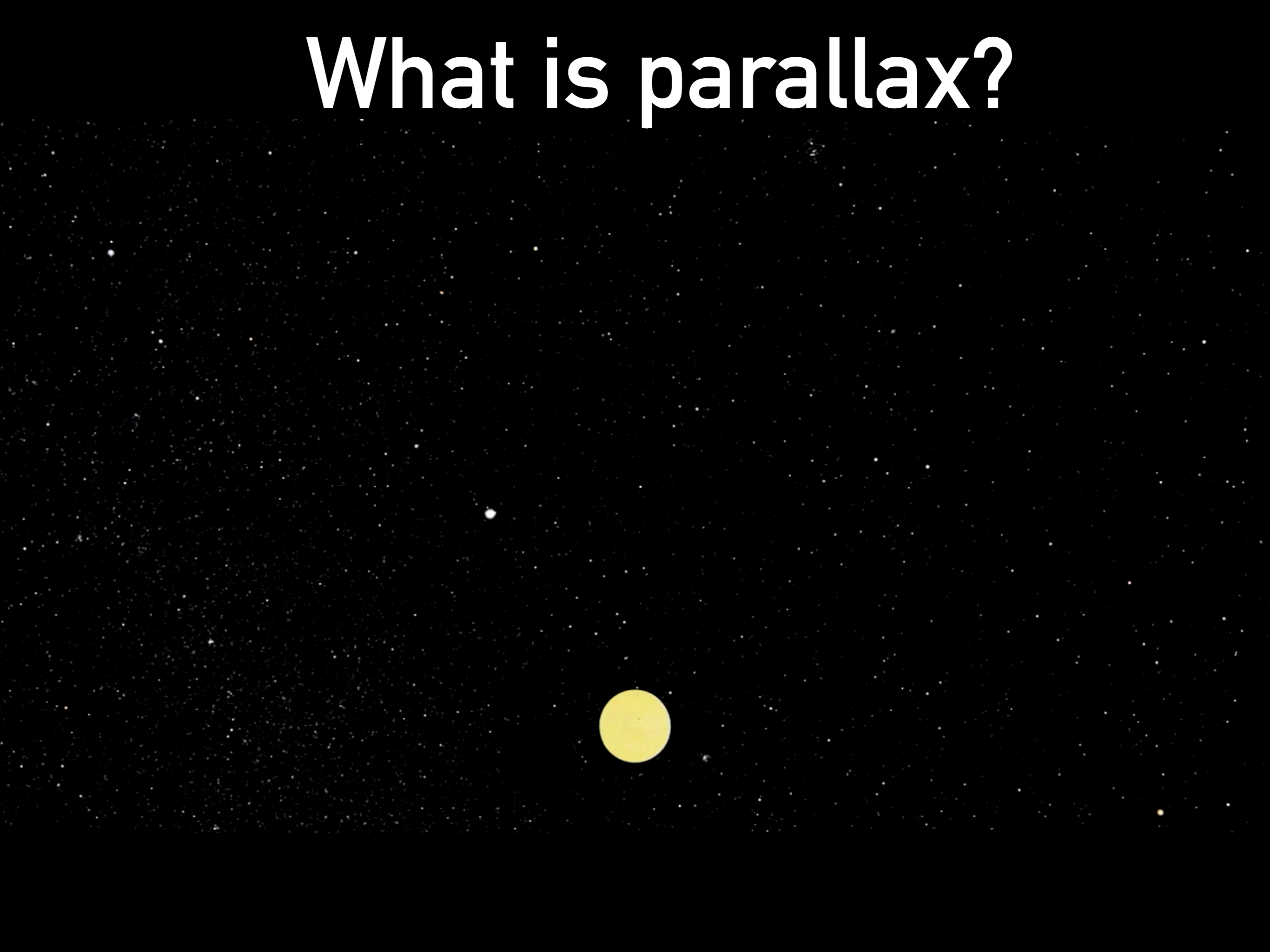
$$P(\boldsymbol{\theta} | \hat{\mathbf{m}}, \hat{\varpi}) \propto \mathcal{L}(\hat{\mathbf{m}} | \boldsymbol{\theta}) \mathcal{L}(\hat{\varpi} | \mu) \pi(\boldsymbol{\theta})$$

**Gaia DR2
Parallax
Measurements**

$$\mathcal{L}(\hat{\varpi} | \mu) = \frac{1}{\sqrt{2\pi}\sigma_{\varpi}} \exp \left[-\frac{1}{2} \frac{(\varpi(\mu) - \hat{\varpi})^2}{\sigma_{\varpi}^2} \right]$$



What is parallax?

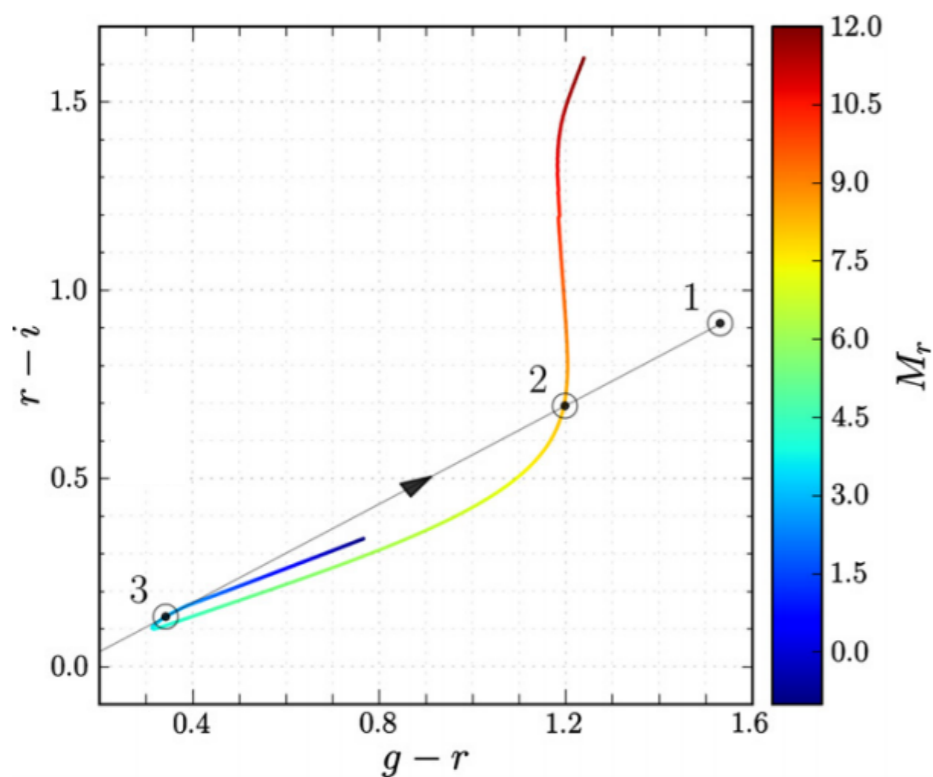


Stellar Inference

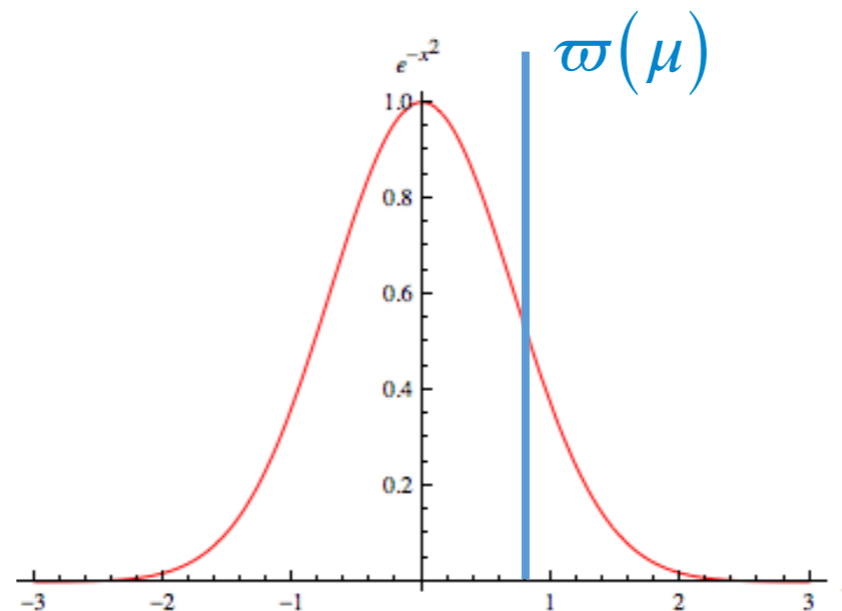
The posterior probability that observed magnitudes $\hat{\mathbf{m}}$ are consistent with our predicted photometry $\mathbf{m}(\theta)$ follows Bayes Rule:

$$\begin{array}{c}
 \text{Posterior} \\
 \boxed{P(\theta | \hat{\mathbf{m}}, \hat{\tau})} \propto \boxed{\text{Photometric Likelihood } \mathcal{L}(\hat{\mathbf{m}} | \theta)} \boxed{\text{Parallax Likelihood } \mathcal{L}(\hat{\tau} | \mu)} \boxed{\text{Priors } \pi(\theta)}
 \end{array}$$

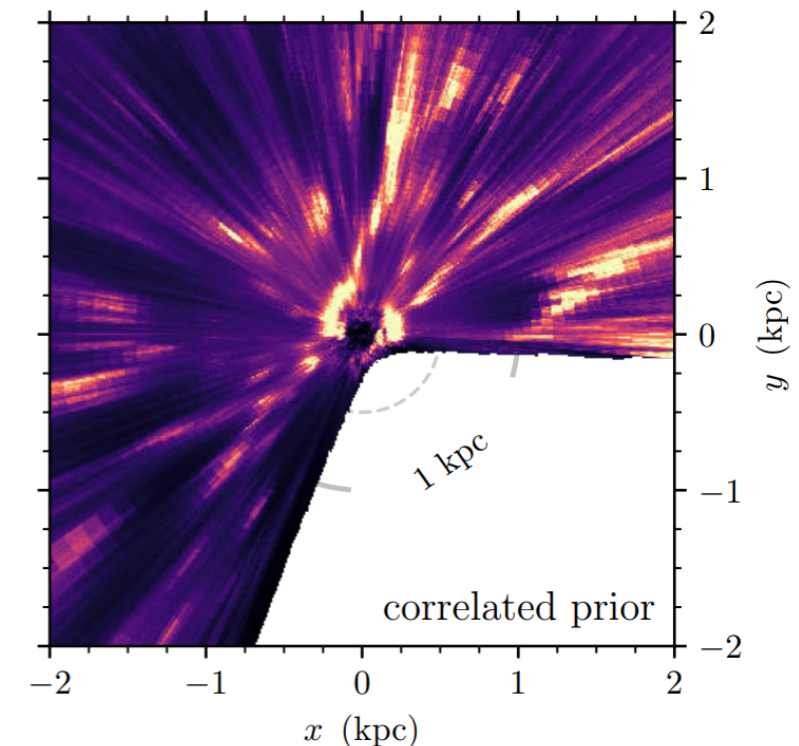
θ : stellar type, reddening, distance



Gaia DR2 Parallax Measurements



3-D Galactic model



Stellar Inference

To derive stellar posteriors we adopt a grid-based approach over a sampling approach. Why?

- Multiple, widely separated solutions
- Posteriors have extended & complex degeneracies (need more samples and/or longer run times than MC methods)

brutus

Speagle et al. (2020a), in prep.

Et tu, Brute?

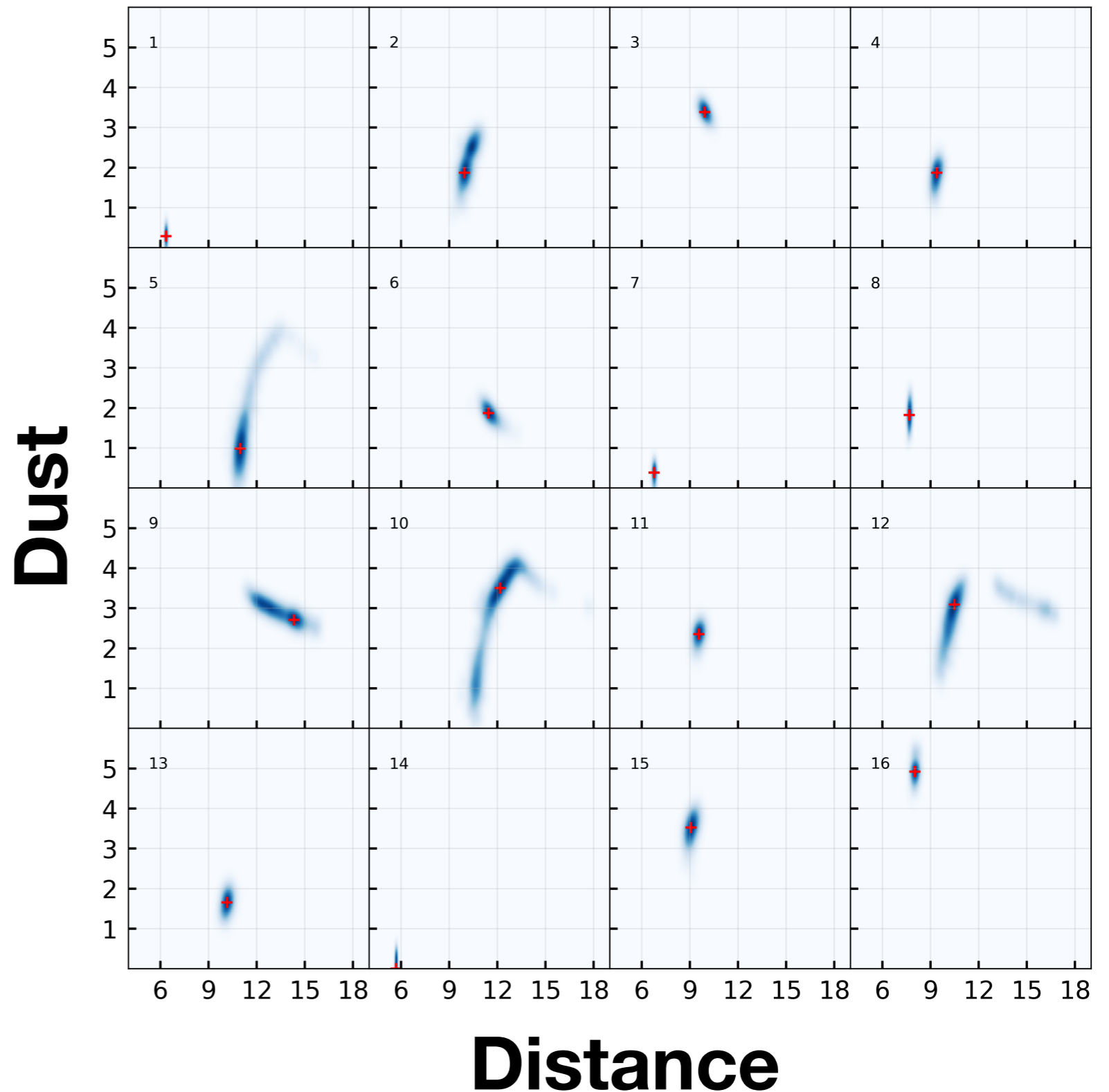
`brutus` is a Pure Python package whose core modules involve using "brute force" Bayesian inference to derive distances, reddenings, and stellar properties from photometry using a grid of stellar models.

The package is designed to be highly modular, with current modules including utilities for modeling individual stars, co-eval stellar associations, and stellar-based 3-D dust mapping.

Per-Star Distance-Dust Posteriors

After Gaia (this Work)

We marginalize over stellar “type” to get posteriors on distance + dust for individual stars



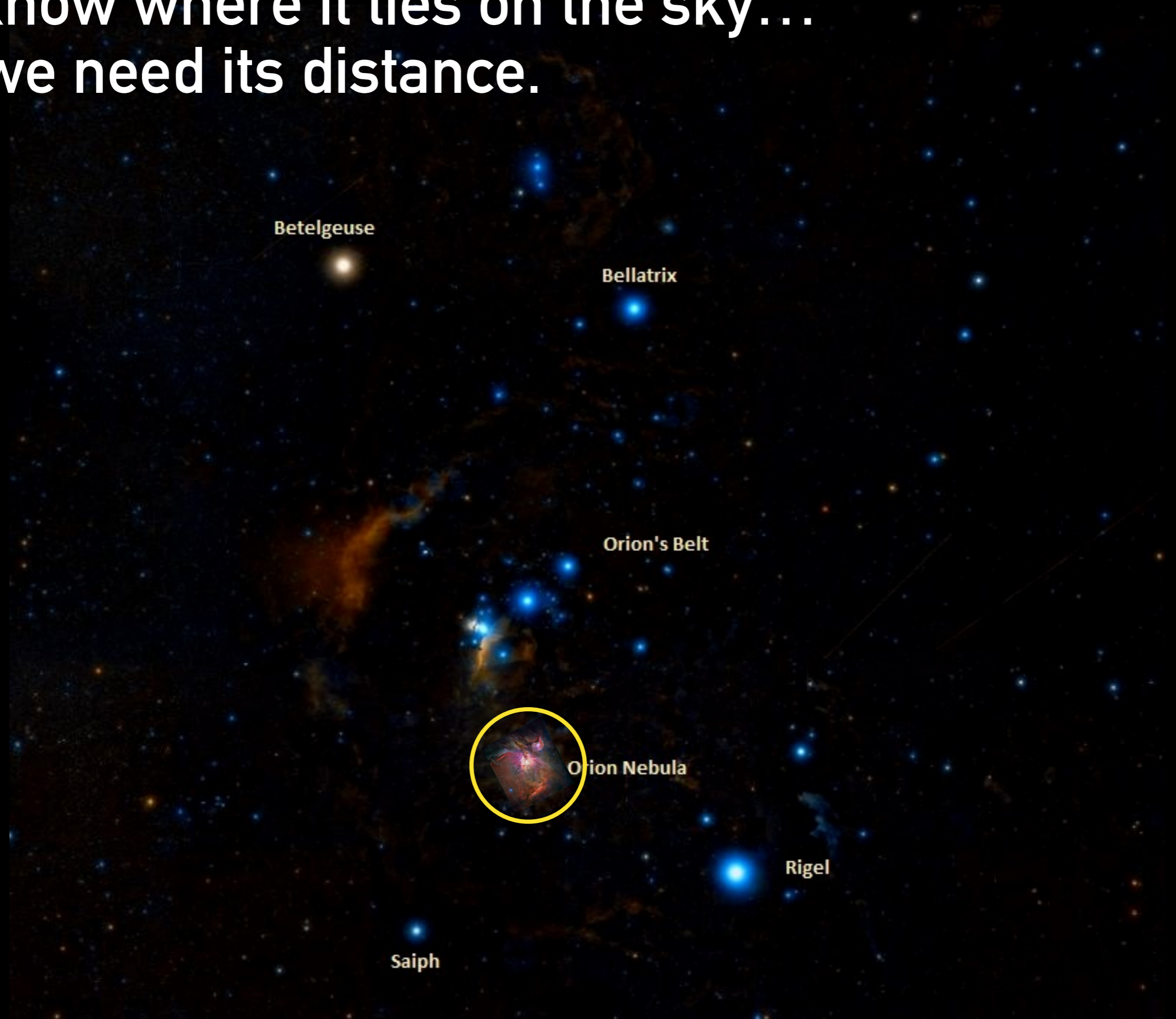
Part 2.

Inferring 3D Dust Cloud Distributions

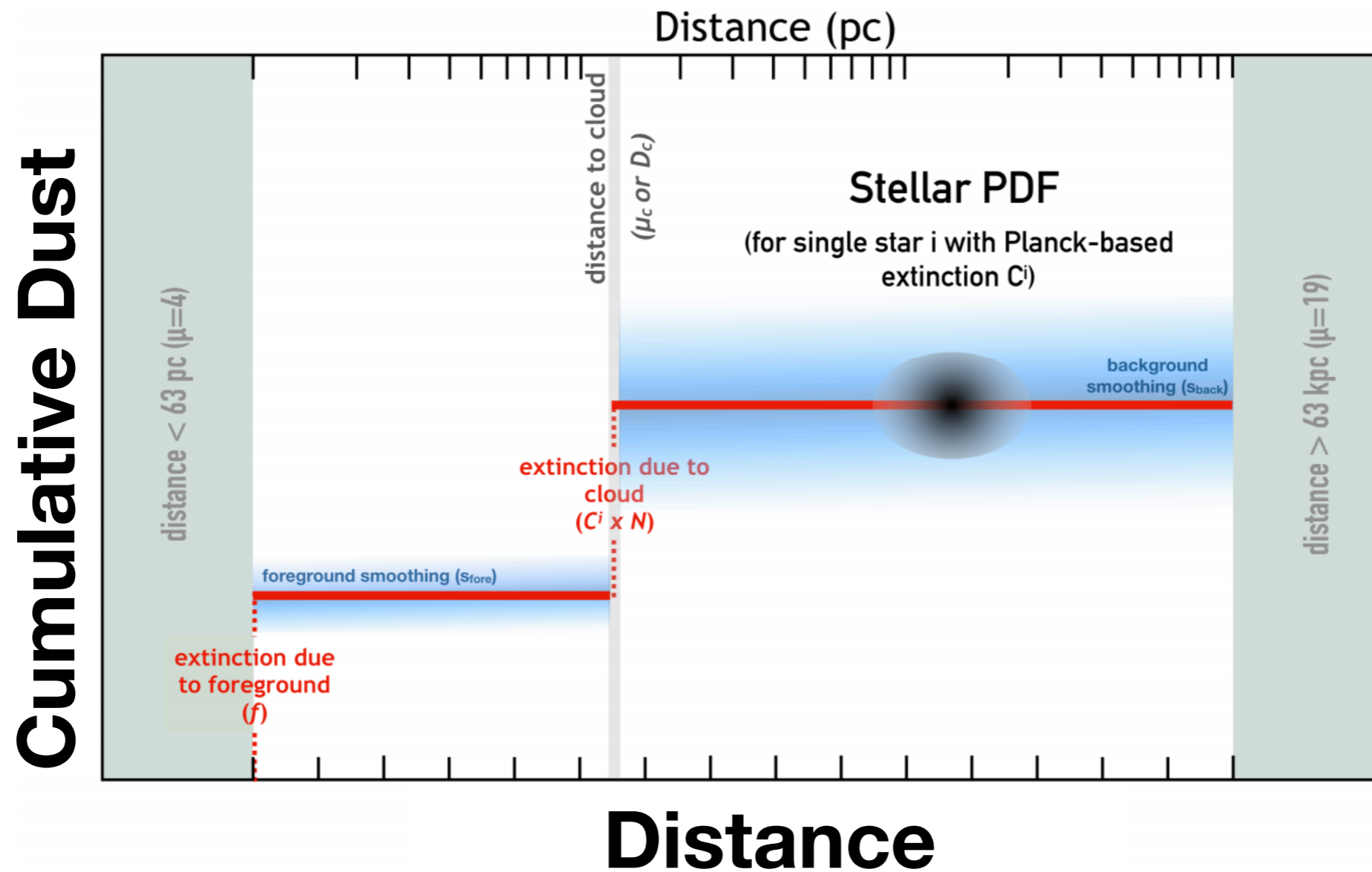
Remember this Nebula?



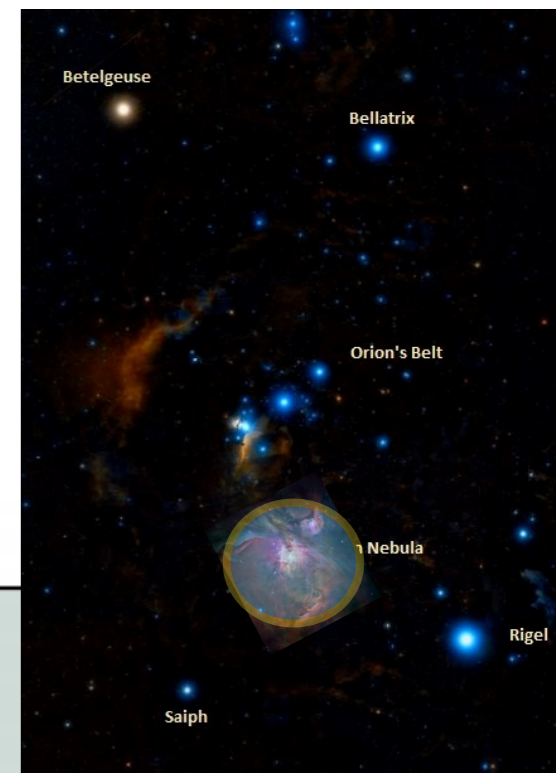
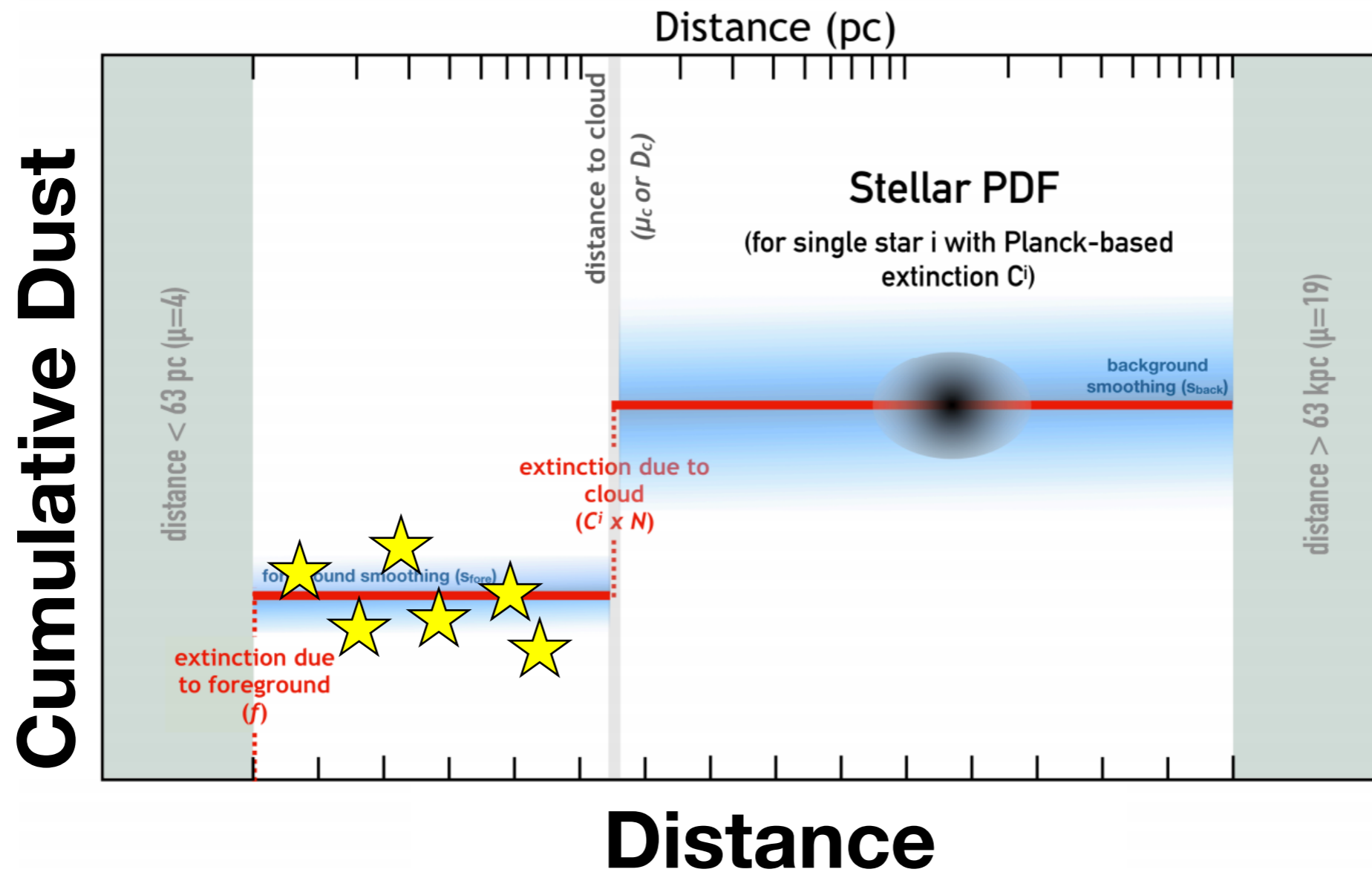
We know where it lies on the sky...
But we need its distance.



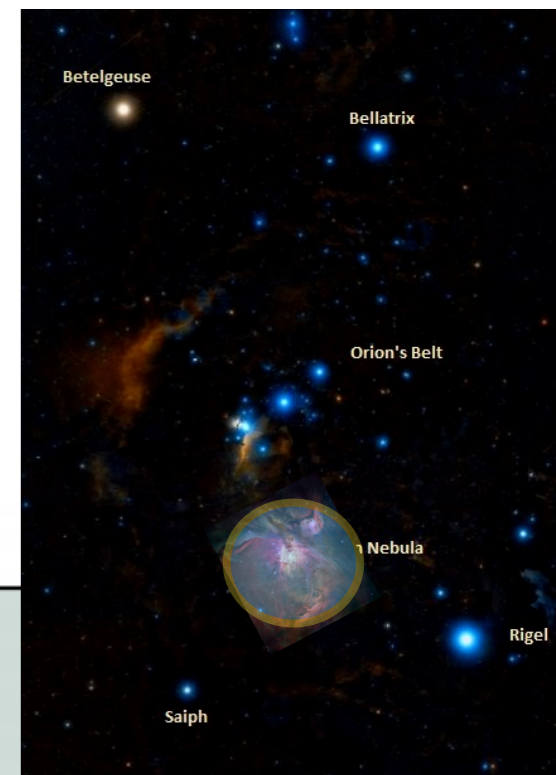
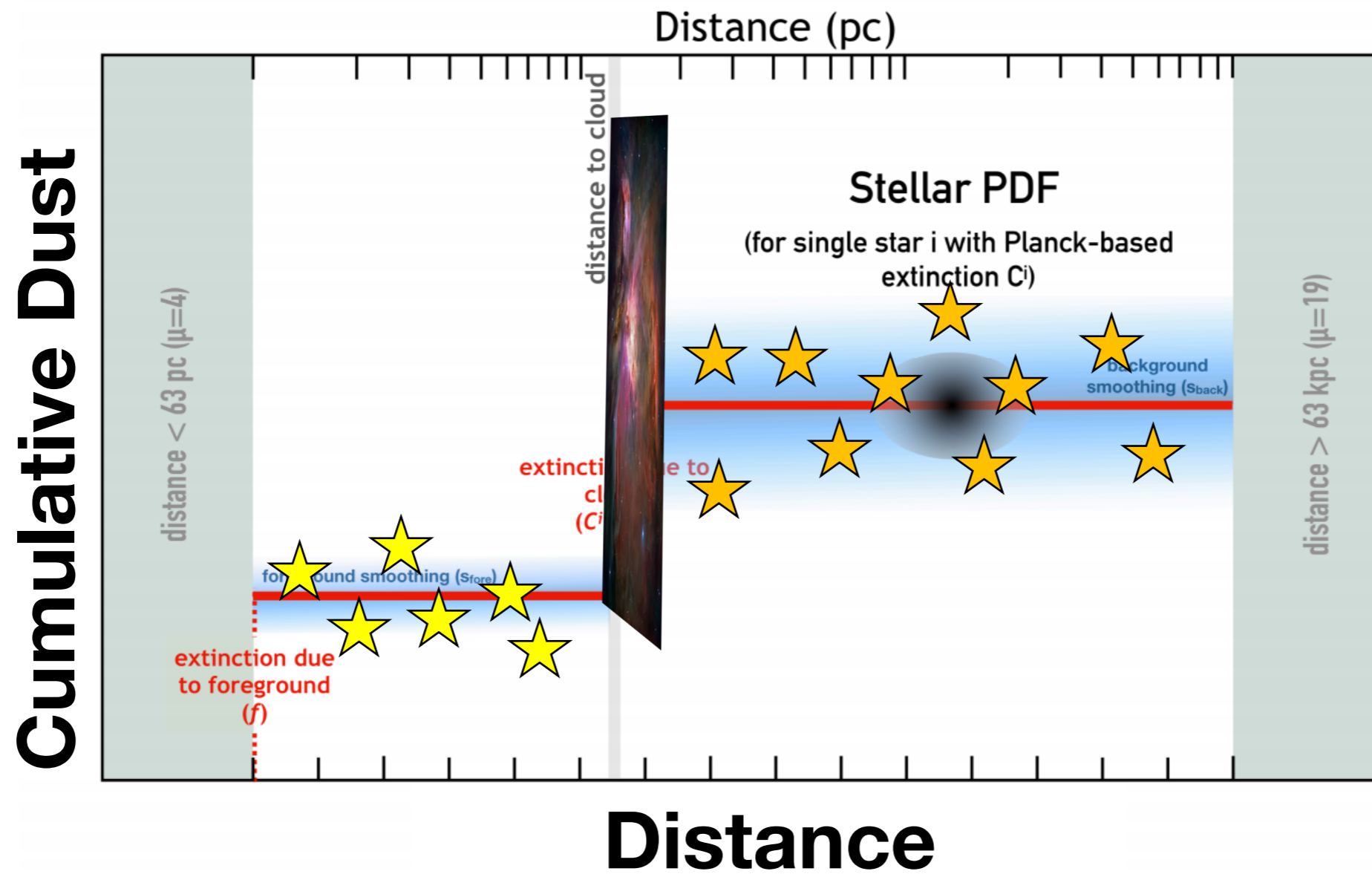
Estimating Cloud Distances



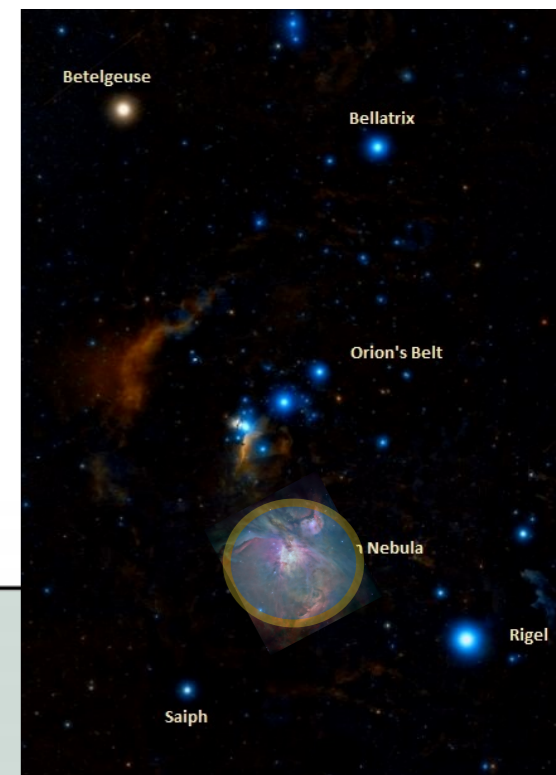
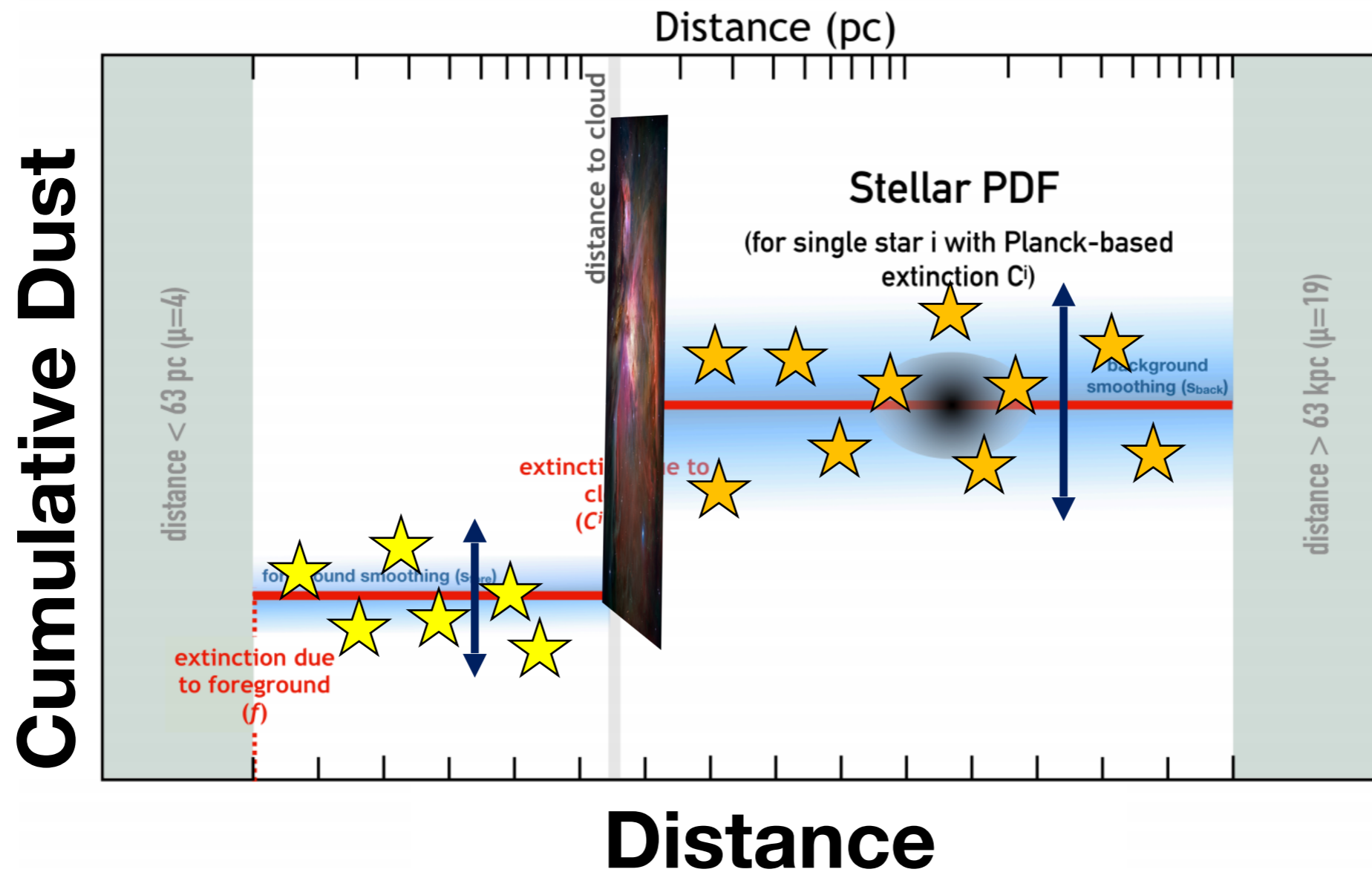
Estimating Cloud Distances



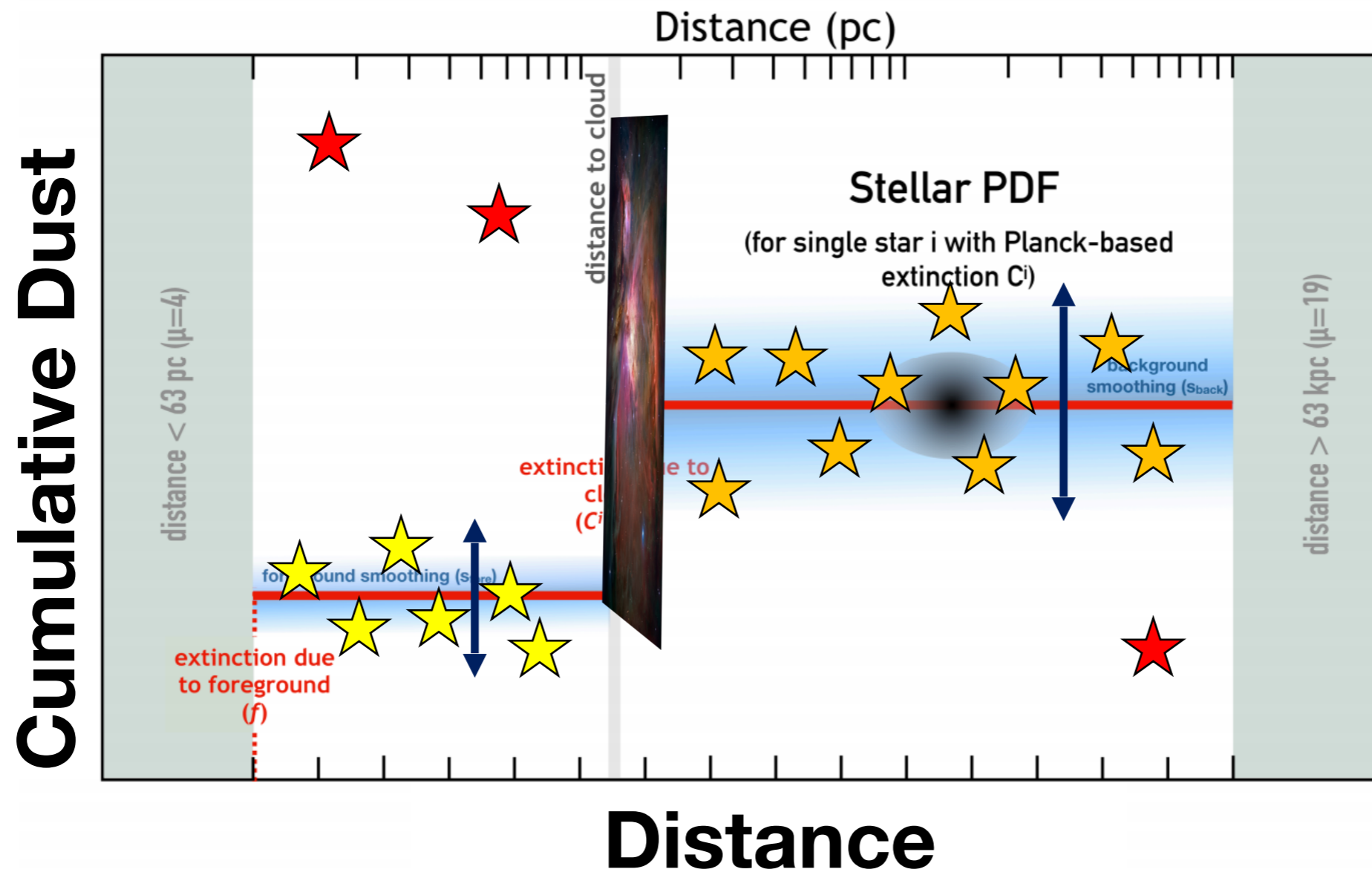
Estimating Cloud Distances



Estimating Cloud Distances



Estimating Cloud Distances



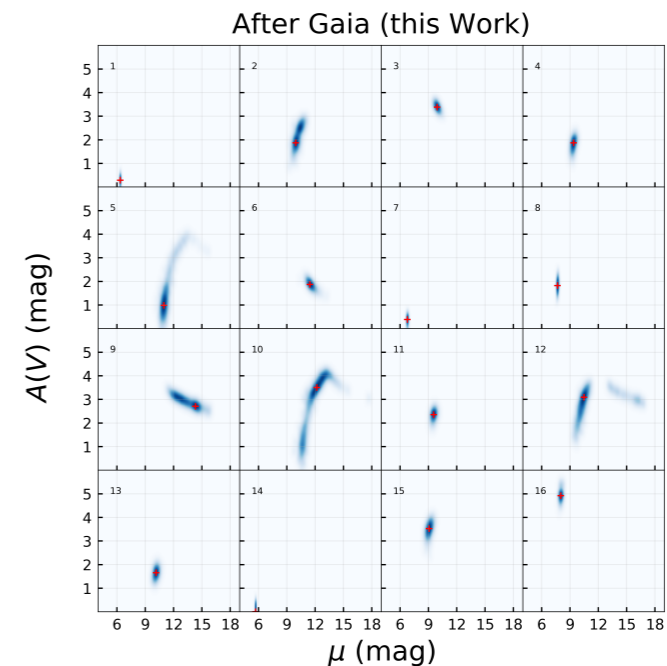
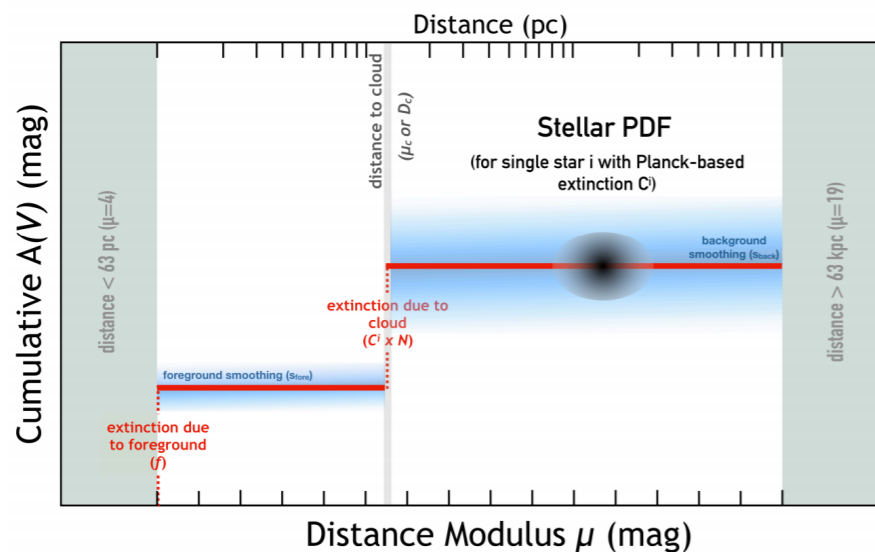
Formalism

Model Parameters

$\alpha = \{d_{\text{cloud}}, \text{ bunch of nuisance parameters}\}$

Line-of-sight dust model
Per-Star Posterior

$$\mathcal{L}(\alpha | \hat{\mathbf{m}}_i) = \int P(\alpha | \mu, A_V) P(\mu, A_V | \hat{\mathbf{m}}_i) d\mu dA_V$$



Formalism

We sample from our six parameter model (cloud distance + 5 nuisance parameters) using **dynamic nested sampler dynesty**



Three main advantages:

1. Can characterize complex uncertainties in real-time (Ferozet et al. 2009).
2. Allocates samples more efficiently (Higson et al. 2017b).
3. Possesses well-motivated stopping criteria (Skilling 2006; Speagle 2020)

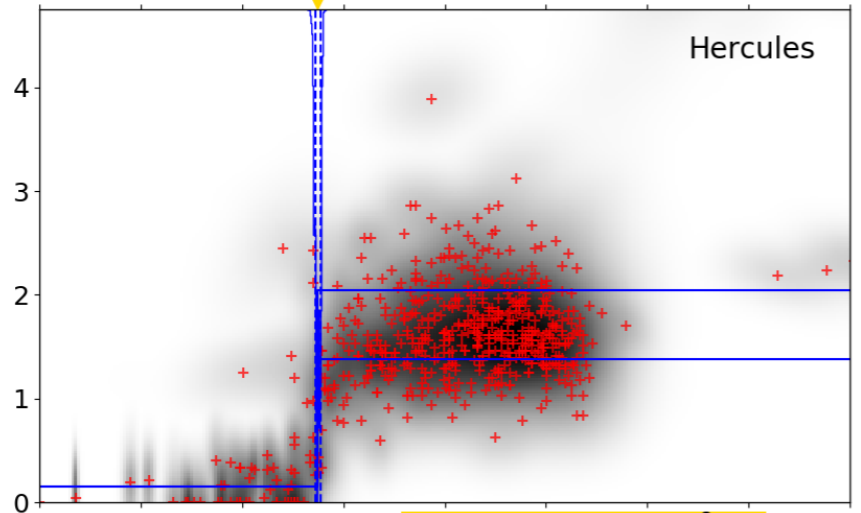
After Gaia (this Work)

D (pc)

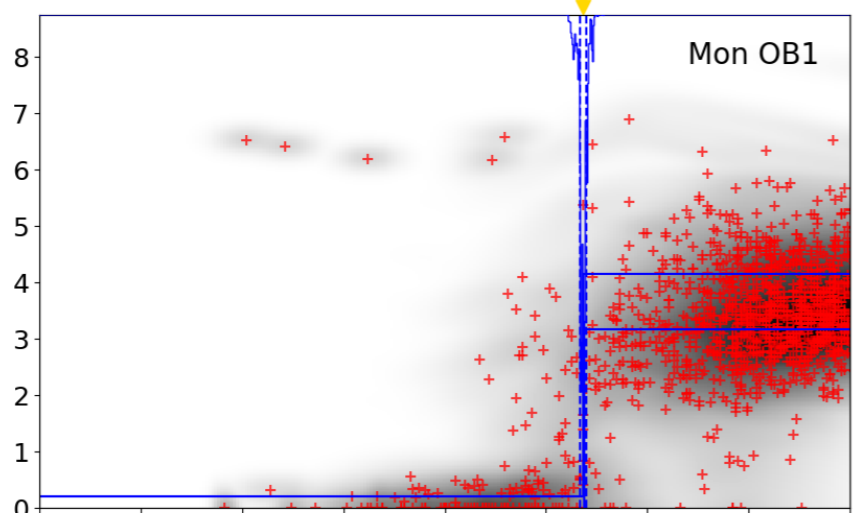
Distance (D_c) = 223^{+2}_{-2} pc

63 100 158 251 398 630 1000 1584 2511

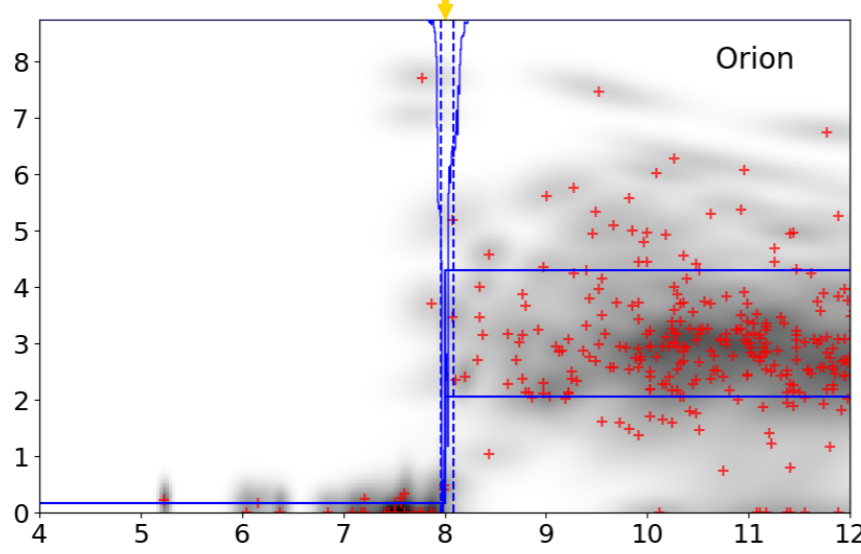
Cumulative Dust



Distance (D_c) = 748^{+9}_{-10} pc

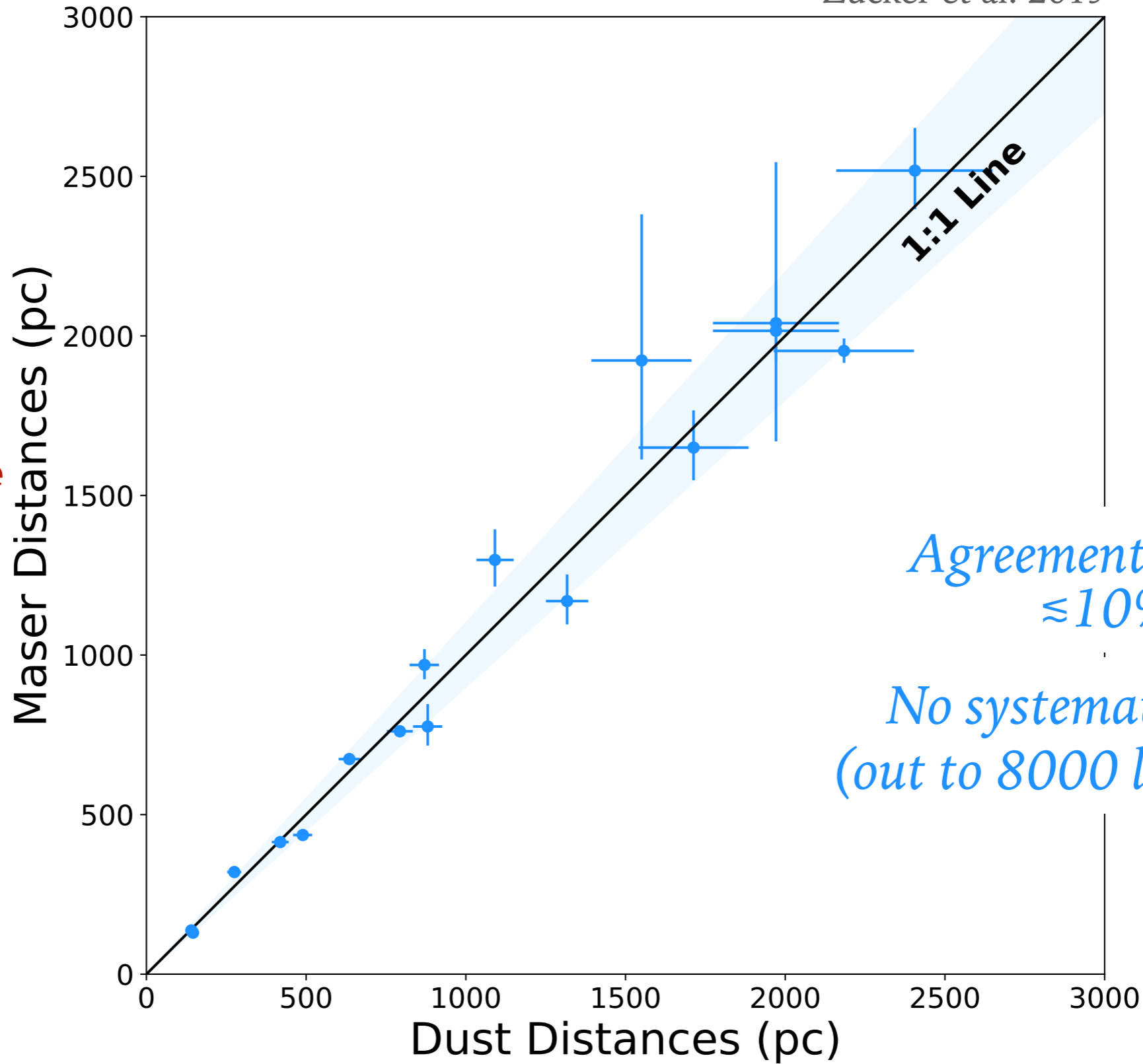


Distance (D_c) = 399^{+14}_{-7} pc

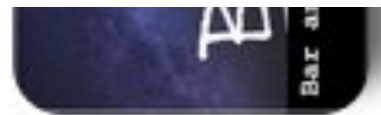


Distance

Results



VERY Expensive
(3000 hrs of
radio
observations)



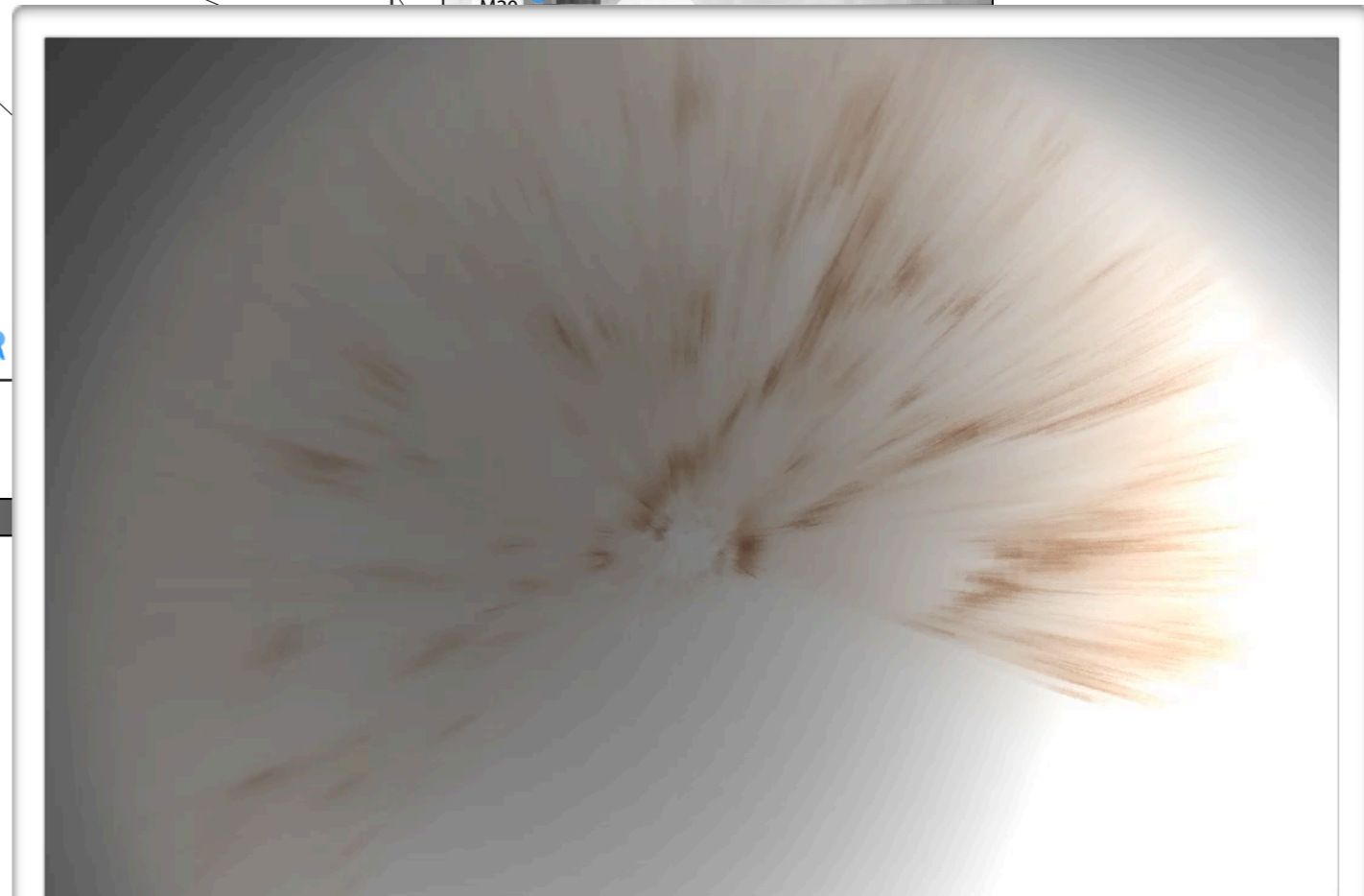
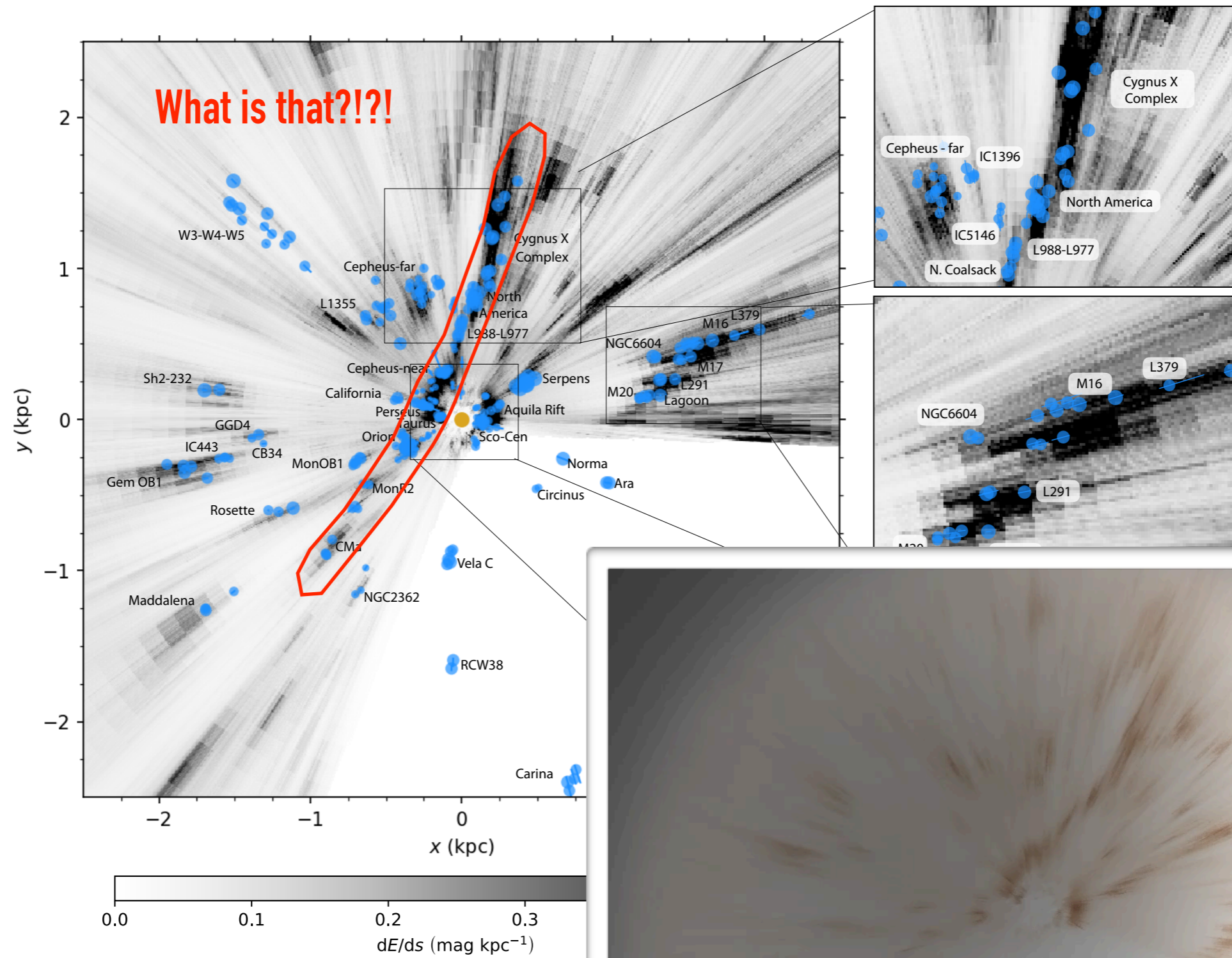
Agreement within
 $\approx 10\%$
No systematic offset
(out to 8000 light years)

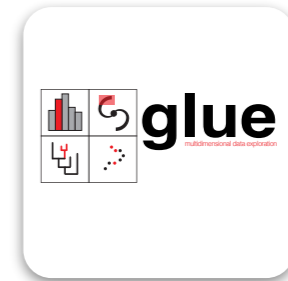
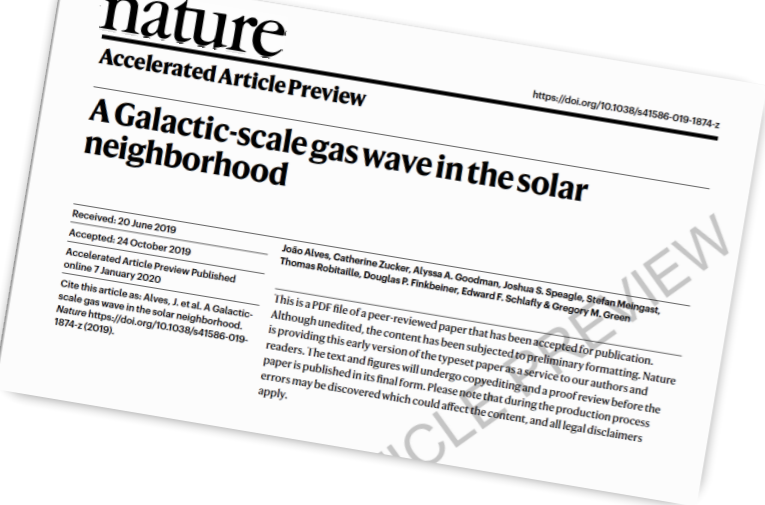
Uses "cheap" publicly
available photometry.
Hours of CPU time

See also Reid+2014, 2016,
Brunthaler+2011, Loinard+2013
Ortiz-Leon+2017a,b, Galli+2018 for
maser/compact radio source references

DISTRIBUTION OF LOCAL CLOUDS

Zucker et al. 2020
Green et al. 2019

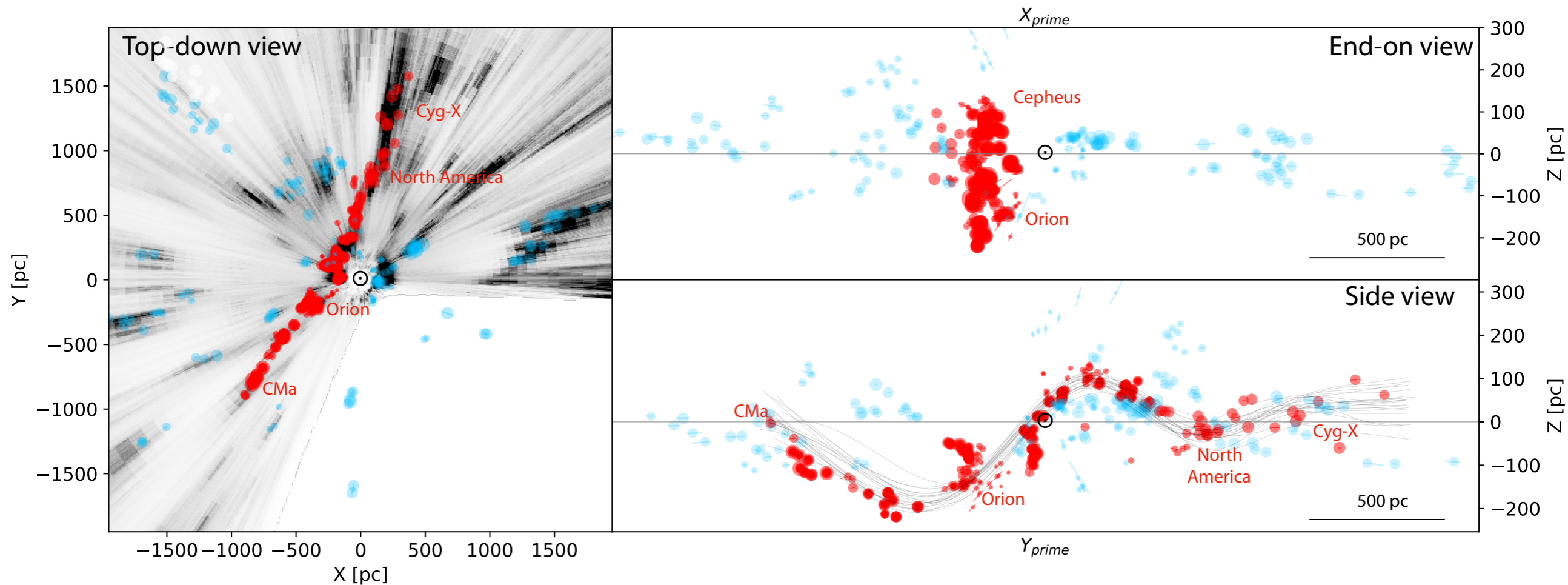




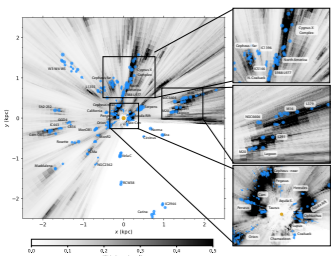
The "Radcliffe" Wave

tinyurl.com/radwave

Alves+2020

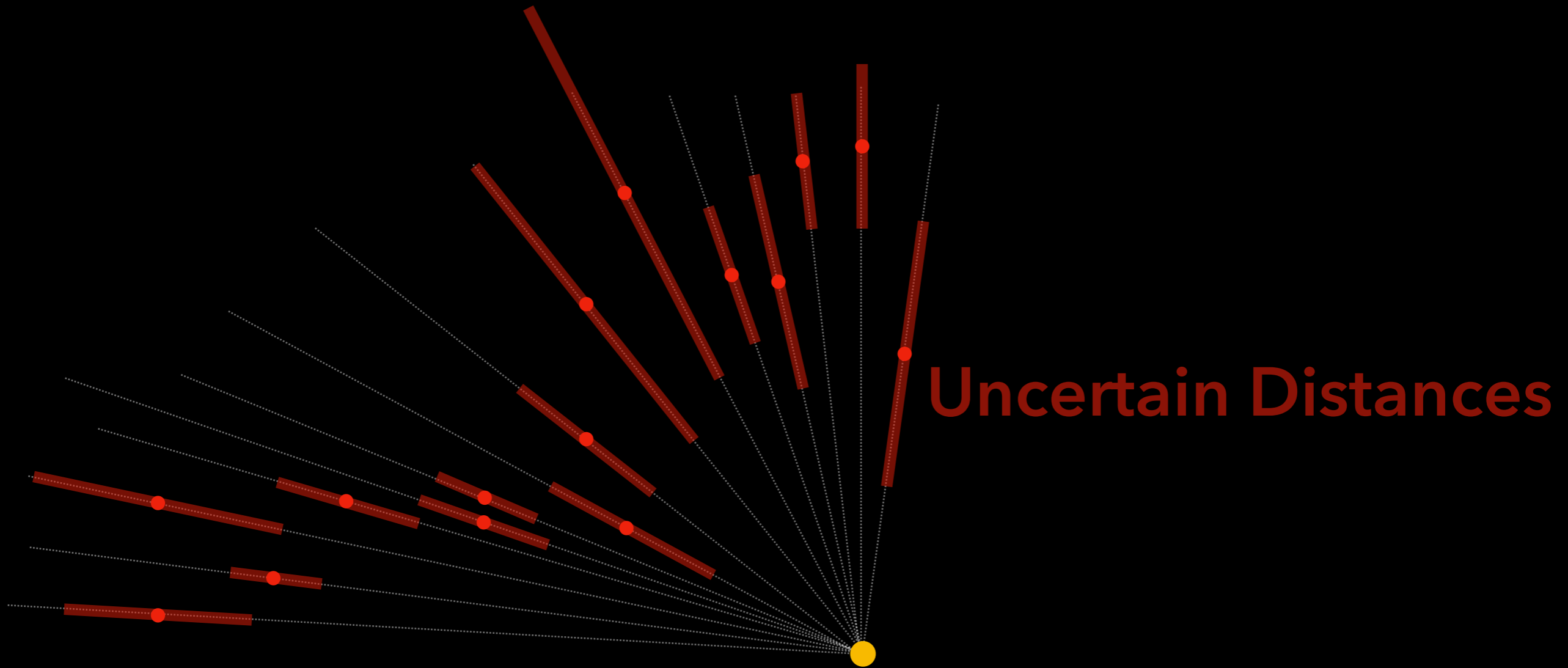


João Alves, Catherine Zucker, Alyssa Goodman, Joshua Speagle, Stefan Meingast, Thomas Robitaille, Douglas Finkbeiner, Edward F. Schlafly, and Gregory Green 2020, **Nature**



Alves et al. 2020

SCHEMATIC CARTOON(!)



Distances estimates **BEFORE** 3D dust mapping & Gaia (~30%)

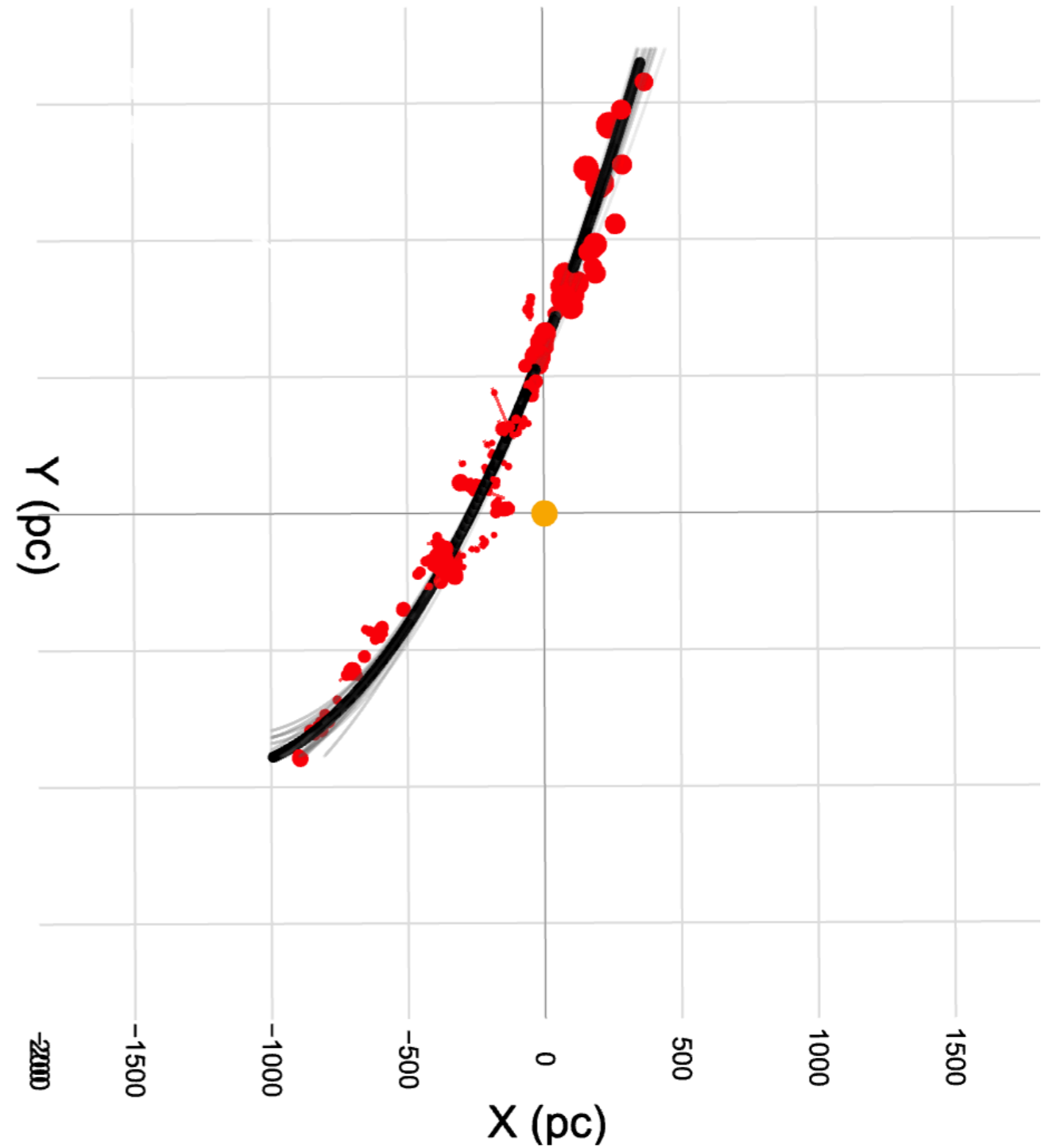
SCHEMATIC CARTOON(!)



Distances estimates **AFTER** 3D dust mapping & Gaia (~5%)

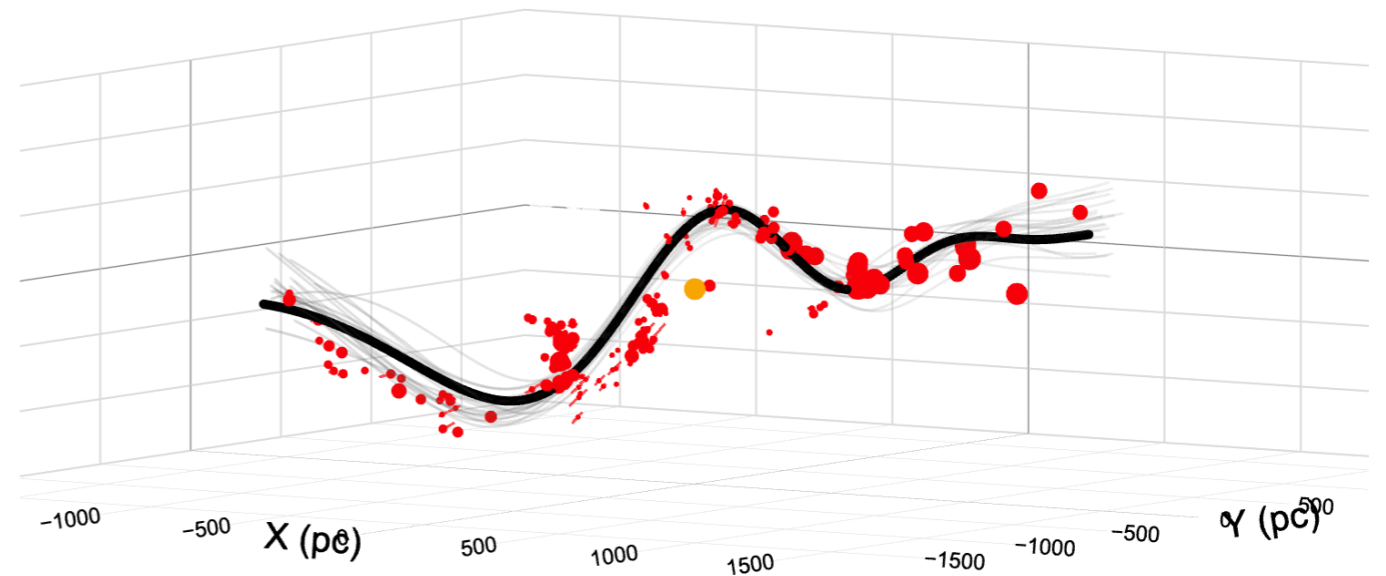
Modeling the Radcliffe Wave

- Model the Radcliffe Wave as a quadratic function in (X, Y, Z) space with respect to three “anchor points”: (x_0, y_0, z_0) , (x_1, y_1, z_1) , (x_2, y_2, z_2)



Modeling the Radcliffe Wave

- Undulating behavior parameterized as a damped sinusoidal function with decaying period and amplitude



$$d(t) = \|(x,y,z)(t) - (x_0,y_0,z_0)\| = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$$

Euclidean distance from start of wave, parameterized by "t"

$$\Delta z(t) = A \times \exp \left[-\delta \left(\frac{d(t)}{\text{kpc}} \right)^2 \right] \times \sin \left[\left(\frac{2\pi d(t)}{P} \right) \left(1 + \frac{d(t)}{\gamma d_{\max}} \right) + \phi \right]$$

Amplitude

Rate of decay of amplitude

Period

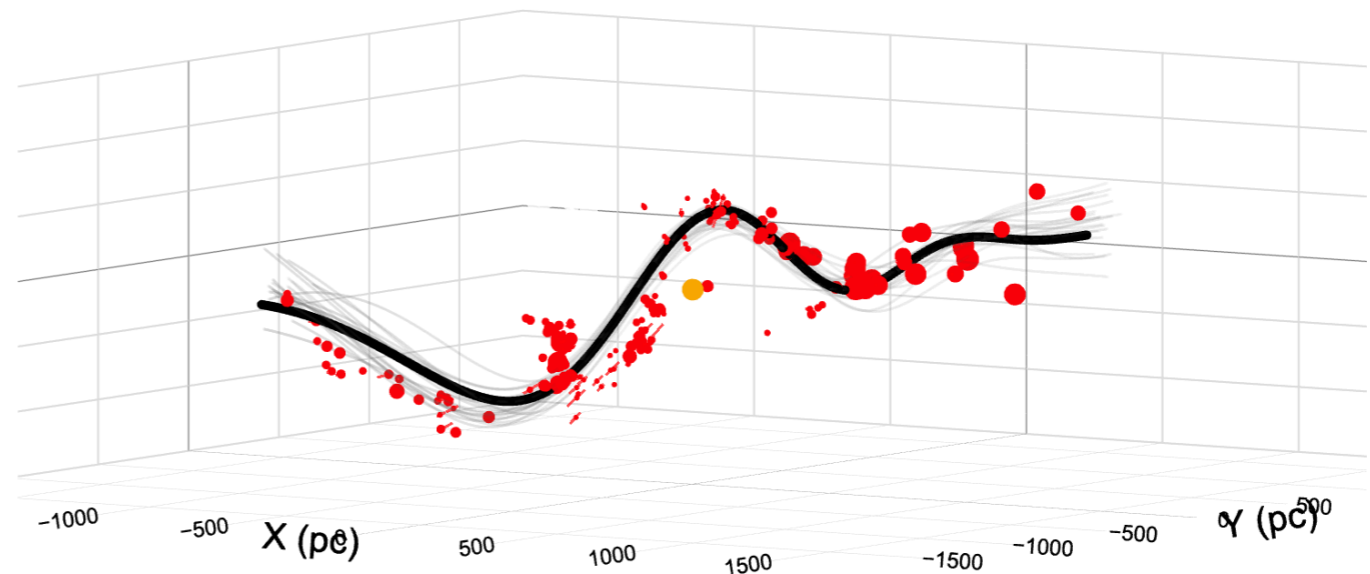
Rate of decay of period

Max distance between start & end of wave

Phase

Modeling the Radcliffe Wave

- Distance of each cloud d_{cloud} (red points) relative to our model is assumed to be normally distributed with some unknown scatter σ :

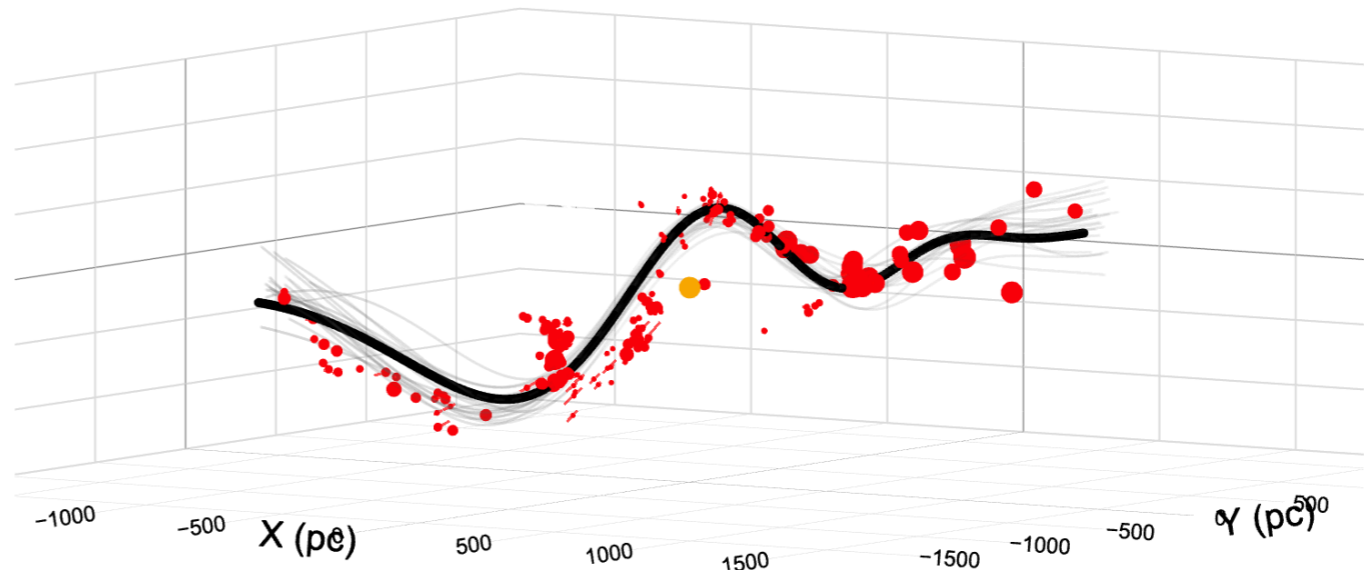


$$d_{cloud} = \min_t (|| (x_{cloud}, y_{cloud}, z_{cloud}) - (x_{wave}, y_{wave}, z_{wave})(t) ||)$$

- We account for structure “off” the Wave by fitting a mixture model.
 - Some fraction f of clouds unassociated with Wave are distributed quasi-uniformly in a large volume around the sun
 - Remaining $1-f$ of clouds associated with Wave

Modeling the Radcliffe Wave

- Likelihood of a realization of our 16-parameter 3D model is given by:



$$\mathcal{L}(\theta) = \prod_{i=1}^n [(1 - f)\mathcal{L}_{\text{cloud},i}(\theta) + f\mathcal{L}_{\text{unif},i}(\theta)] \quad (3)$$

where

$$\mathcal{L}_{\text{cloud},i}(\theta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2} \frac{d_{\text{cloud},i}^2}{\sigma^2}\right], \quad \mathcal{L}_{\text{unif},i}(\theta) = 10^{-7}$$

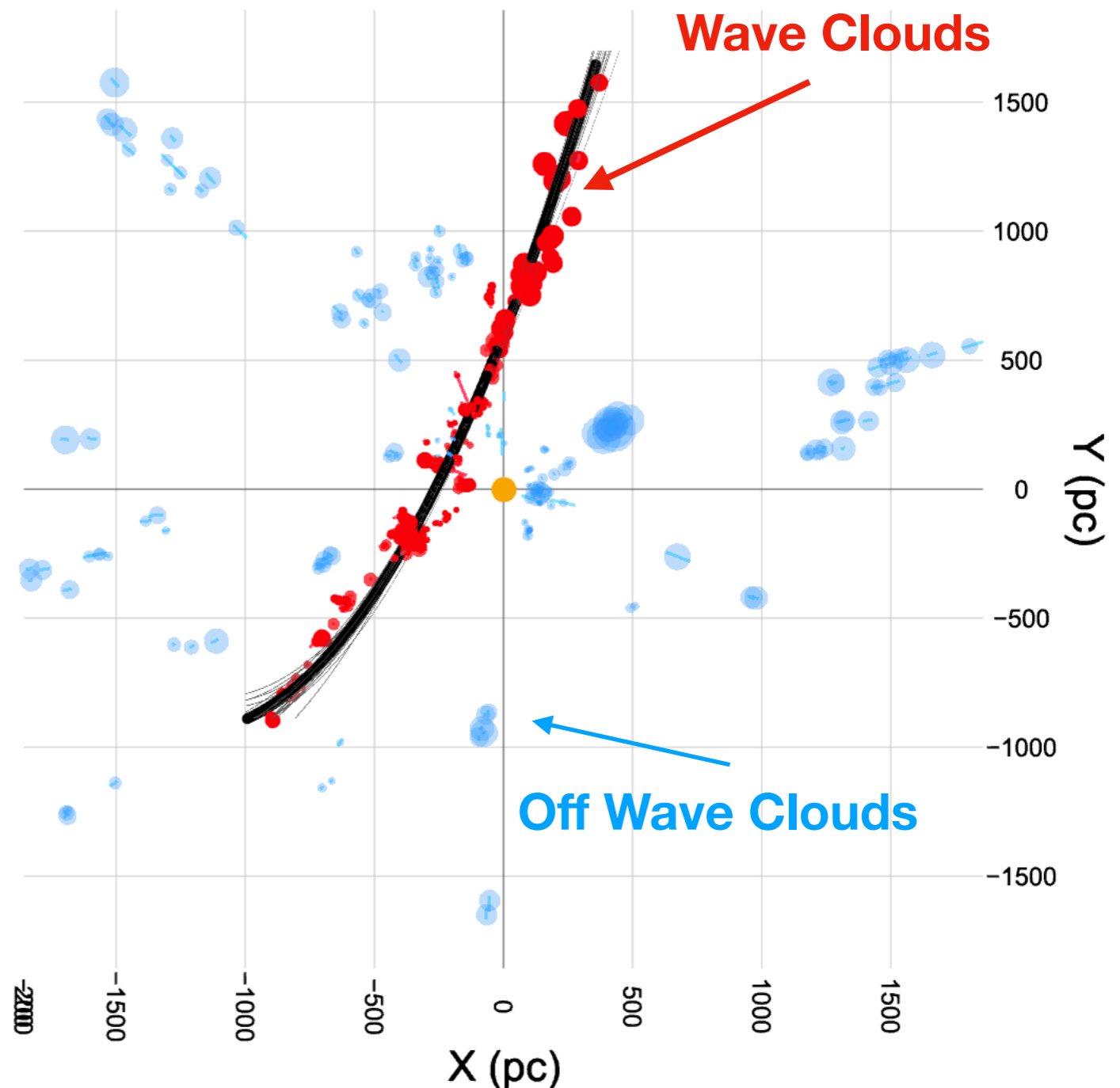
- Generate samples from posterior using the nested sampling code dynesty (Speagle 2020)

Modeling the Radcliffe Wave

- Using our samples, we associate particular clouds with the Wave by computing the mean odds ratio averaged over the posterior:

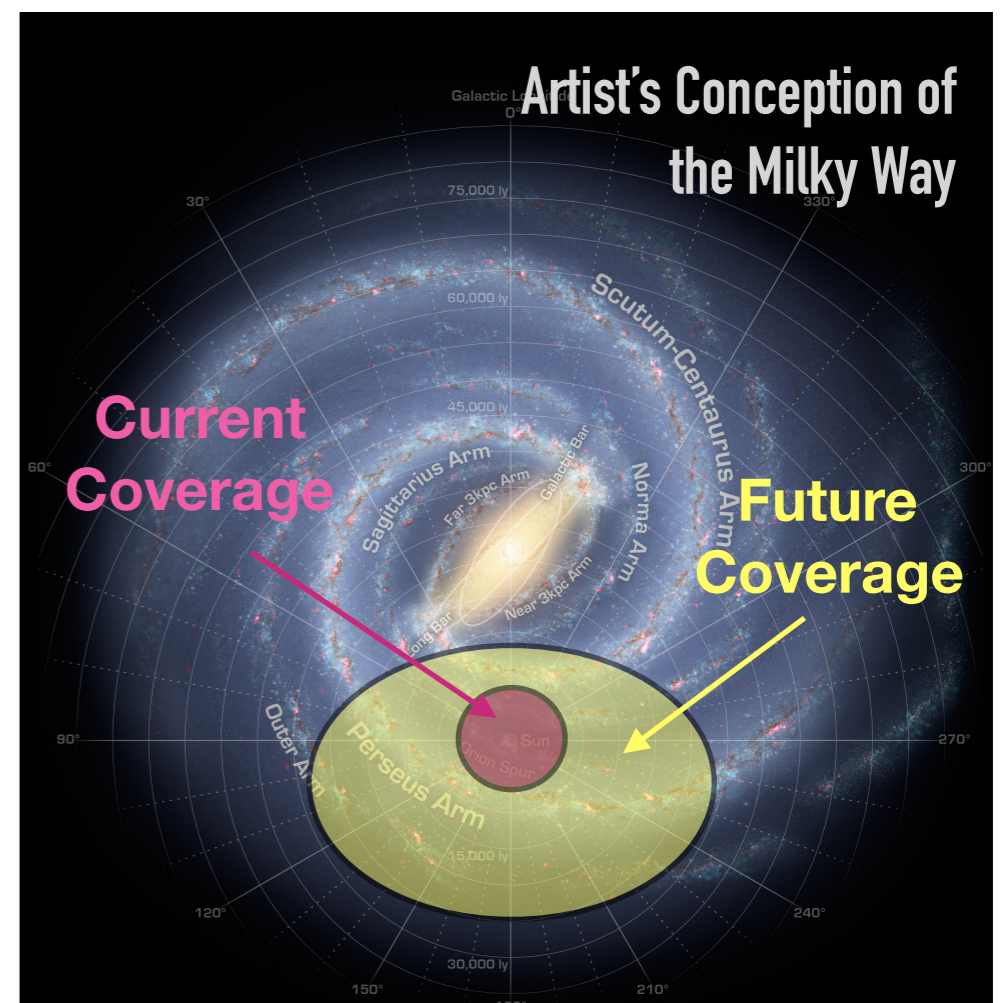
$$\langle R_i \rangle = \int \frac{(1 - f) \mathcal{L}_{\text{cloud},i}(\theta)}{f \mathcal{L}_{\text{unif},i}(\theta)} \mathcal{P}(\theta) d\theta$$

- We classify all clouds with $\langle R_i \rangle > 1$ as being part of the Wave



The Future...

- Update our stellar modeling pipeline (switch from "empirical" models to "theoretical" models) to so we can see through more dust at farther distances
- Only using a small fraction of the available photometry (~ 1 billion out of ~ 5 billion stars). Incorporate more data, at more wavelengths!
- Star and dust modeling is currently decoupled. These properties should be jointly estimated in the context of a hierarchical model
 - (block-)Gibbs schemes?
 - Importance resampling?
- Your ideas here...



Thanks! Any (more?) questions?