Hierarchical Bayesian method for constraining the neutron star equation of state with an ensemble of binary neutron star postmerger remnants: statistical, computational, and collaborative challenges

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How it began

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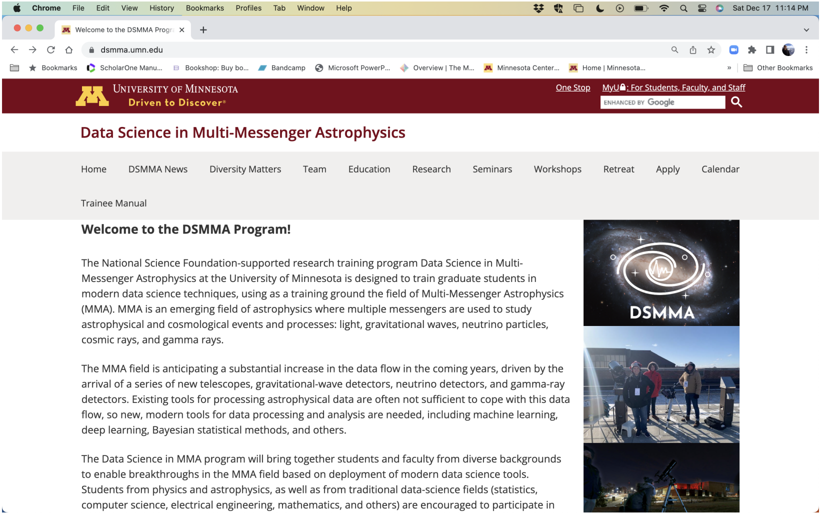
My research focuses on the physics of the earliest stages of the evolution of the universe and of the correspondingly high energies that cannot be reproduced in laboratories. I work on experiments that probe the content and properties of the Universe today: LIGO, which aims to measure gravitational waves generated by various events and processes in the universe; and SuperCDMS, which aims to detect dark matter in the form of new weakly interactive massive particles.

My group uses Bayesian inference to extract information about the gravitational wave sky from data acquired by LIGO detectors. This includes imaging the gravitational wave sky and estimating the frequency, directionality, and polarization properties of the gravitational wave background on the sky. We use similar techniques to study implications of dark matter searches for many-dimensional parameter space of fundamental particle models. We also use machine learning techniques for removing environmental contamination from gravitational-wave data and to optimally extract information from dark matter cryogenic semiconductor detectors.

Data Science in Multi-Messenger Astrophysics



DSMMA Program

Above all else it is **interdisciplinary and collaborative**

Physics & Astronomy, Statistics, Electrical Engineering, and Computer Science

Team-taught courses in Bayesian Astrostatistics and Machine Learning for Astrophysics

Research project student teams mentored by interdisciplinary teams of faculty

Research Projects

Since 2021 I have helped mentor four projects:

(i) a study of supernova siblings and the properties of the host galaxies

(ii) using observed kilonova candidates to inform ejecta quantities

(iii) cross-correlation between stochastic gravitational wave backgrounds and the cosmic microwave background

(iv) **constraining the neutron star equation of state based on binary neutron star inspiral and post-merger gravitational wave signals**.

Constraining the Neutron Star EoS

EoS – relationship between NS mass and radius (or, equivalently, pressure and density)

Goal is to constrain $R_{1.6}$, the radius of a $1.6 M_{\odot}$ NS

Current consensus: $R_{1.6} = 12.07^{+0.98}_{-0.77}$ km (95% credible interval)

Our work: $R_{1.6} = 11.91^{+0.80}_{-0.56}$ km (95% credible interval)

NS EoS: Statistical Formalism and Challenges

Goal: Constrain $R_{1.6}$

Inspiral signal provides the chirp mass

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

Post-merger signal peak frequency, f_{peak}.

Structure of Bayesian model:

$$\begin{aligned} D_i \mid \mathcal{M}_i, f_{\text{peak},i} &\sim p(D_{IN,i} \mid \mathcal{M}_i) p(D_{PM,i} \mid f_{\text{peak},i}) \\ \mathcal{M}_i, f_{\text{peak},i} \mid R_{1.6} &\sim \nu(\mathcal{M}_i, f_{\text{peak},i} \mid R_{1.6}) \\ R_{1.6} &\sim \nu(R_{1.6}) \end{aligned}$$

Likelihood

$$
\begin{aligned}
p(D|R_{1.6}) &= \prod_{i=1}^{N} p((D_{\mathrm{PM}}, D_{\mathrm{IN}})_i | R_{1.6}) \\
&= \prod_{i=1}^{N} \int p(D_{\mathrm{IN},i}, D_{\mathrm{PM},i} | \mathcal{M}_i, f_{\mathrm{peak},i}) \\
&\qquad \times \nu(\mathcal{M}_i, f_{\mathrm{peak},i} | R_{1.6}) df_{\mathrm{peak},i} d\mathcal{M}_i \\
&= \prod_{i=1}^{N} \int p(D_{\mathrm{IN},i} | \mathcal{M}_i) p(D_{\mathrm{PM},i} | f_{\mathrm{peak},i}) \\
&\qquad \times \nu(\mathcal{M}_i, f_{\mathrm{peak},i} | R_{1.6}) df_{\mathrm{peak},i} d\mathcal{M}_i,
\end{aligned}
$$

Complication

$D_i = (D_{IN,i}, D_{PM,i})$ is not available.

Instead the Bilby pipeline provides samples from

$$q(\mathcal{M}_i \mid D_{IN,i}) \propto p(D_{IN,i} \mid \mathcal{M}_i) p_0(\mathcal{M}_i)$$

and the BayesWave pipeline provides samples from

$$q(f_{\text{peak},i} \mid D_{PM,i}) \propto p(D_{PM,i} \mid f_{\text{peak},i}) p_1(f_{\text{peak},i})$$

If BayesWave does not detect a peak frequency, then samples from $p_1(f_{\text{peak}})$ are returned.

Likelihood

$$p(D|R_{1.6}) = \prod_{i=1}^{N} \int \frac{p(\mathcal{M}_i|D_{\mathrm{IN},i})p(D_{\mathrm{IN},i})}{p_0(\mathcal{M}_i)} \frac{p(f_{\mathrm{peak},i}|D_{\mathrm{PM},i})p(D_{\mathrm{PM},i})}{p_1(f_{\mathrm{peak},i})} \times \nu(f_{\mathrm{peak},i}, \mathcal{M}_i \mid R_{1.6}) df_{\mathrm{peak},i} d\mathcal{M}_i$$

The likelihood is not available in analytic form.

Instead we have a Monte Carlo approximation of it from the Bilby and BayesWave posterior samples.

How good is the approximation?

Open Question

The BayesWave pipeline which is used to sample from

$$q(f_{\text{peak},i} \mid D_{PM,i}) \propto p(D_{PM,i} \mid f_{\text{peak},i}) p_1(f_{\text{peak},i})$$

uses reversible jump Metropolis-Hastings to produce the samples.

RJMH is not at all well understood and has a reputation for being finicky and unreliable.

Is the BayesWave pipeline reliable?

The theory required to study this question is being developed.

Priors

For $R_{1.6}$ we consider two options

$$R_{1.6} \sim \text{Uniform}(9\text{km}, 15\text{km})$$

and an astrophysical prior which is the posterior reported by Huth et al (2022).

For $\mathcal{M}$ we use priors that match values found by Abbott et al (2020) and Petrov et al (2022) which yield

$$\mathcal{M} \sim N(1.33M_\odot, 0.09M_\odot)$$

Priors

Vretinaris et al (2020) use numerical relativity simulations to derive EoS-agnostic relations between f_{peak} and $\mathcal{M}$ of the form

$$\frac{f_{\text{peak}}}{\mathcal{M}} = g(\mathcal{M}, R_{1.6}) + \varepsilon \qquad \varepsilon \sim N(0, \sigma^2)$$

but we need something of the form

$$f_{\text{peak}} = g(\mathcal{M}, R_{1.6}) + \varepsilon \qquad \varepsilon \sim N(0, \sigma^2)$$

Prior

Vretinaris et al (2020) suggests f_{peak} should be centered around

$$\mu_f = \beta_0 + \beta_1 \mathcal{M} + \beta_2 \mathcal{M}^2 + \beta_3 R_{1.6} \mathcal{M} + \beta_3 R_{1.6} \mathcal{M}^2 + \beta_3 R_{1.6}^2 \mathcal{M}$$

Formally,

$$f_{\text{peak}} \mid \text{data}, \mu_f, \lambda_f \sim N(\mu_f, \lambda_f^{-1})$$
$$\nu(\mu_f, \lambda_f) = \lambda_f^{-1/2}$$

yielding a posterior

$$q(\mu_f \mid \text{data})$$

We also consider an empirical Bayes approach to prewhitening and set

$$f_{\text{peak}} = E[\mu_f \mid \text{data}]$$

Model Validation

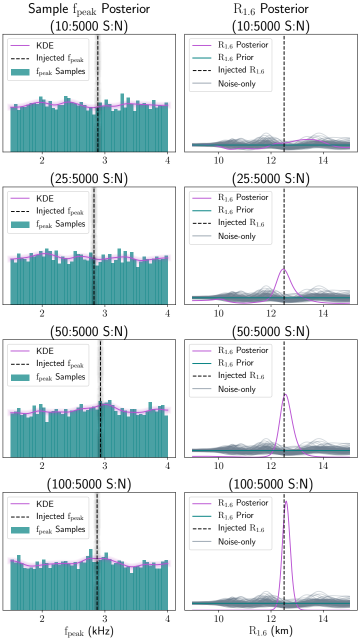
Validation: What happens if post-merger signals are just noise?

Developed a novel hypothesis testing framework based on Wasserstein distance.

Model Validation: Perform tests under ideal conditions by

- simulating BayesWave posteriors for f_{peak}
- using wide range of merger masses
- using a variety of EoSs

Ensemble of Mergers



Model Evaluation

Simulations of future observing runs:

- simulate 2 one-year observing runs of LVK (O4, O5)
- Use two different EoSs

Simulation of Future Runs