

Hierarchical Bayesian method for constraining the  
neutron star equation of state with an ensemble  
of binary neutron star postmerger remnants:  
statistical, computational, and collaborative  
challenges

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# How it began



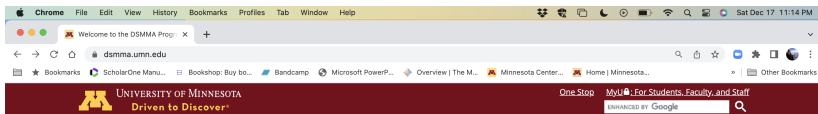
## Vuk Mandic

Professor, School of Physics and Astronomy and Minnesota Institute for Astrophysics

My research focuses on the physics of the earliest stages of the evolution of the universe and of the correspondingly high energies that cannot be reproduced in laboratories. I work on experiments that probe the content and properties of the Universe today: LIGO, which aims to measure gravitational waves generated by various events and processes in the universe; and SuperCDMS, which aims to detect dark matter in the form of new weakly interactive massive particles.

My group uses Bayesian inference to extract information about the gravitational wave sky from data acquired by LIGO detectors. This includes imaging the gravitational wave sky and estimating the frequency, directionality, and polarization properties of the gravitational wave background on the sky. We use similar techniques to study implications of dark matter searches for many-dimensional parameter space of fundamental particle models. We also use machine learning techniques for removing environmental contamination from gravitational-wave data and to optimally extract information from dark matter cryogenic semiconductor detectors.

# Data Science in Multi-Messenger Astrophysics



The screenshot shows a Chrome browser window with the URL [dsmma.umn.edu](http://dsmma.umn.edu). The page header features the University of Minnesota logo and the slogan "Driven to Discover". Navigation links include "Home", "DSMMA News", "Diversity Matters", "Team", "Education", "Research", "Seminars", "Workshops", "Retreat", "Apply", and "Calendar". A search bar is present with the text "ENHANCED BY Google".

## Data Science in Multi-Messenger Astrophysics

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### Welcome to the DSMMA Program!

The National Science Foundation-supported research training program Data Science in Multi-Messenger Astrophysics at the University of Minnesota is designed to train graduate students in modern data science techniques, using as a training ground the field of Multi-Messenger Astrophysics (MMA). MMA is an emerging field of astrophysics where multiple messengers are used to study astrophysical and cosmological events and processes: light, gravitational waves, neutrino particles, cosmic rays, and gamma rays.

The MMA field is anticipating a substantial increase in the data flow in the coming years, driven by the arrival of a series of new telescopes, gravitational-wave detectors, neutrino detectors, and gamma-ray detectors. Existing tools for processing astrophysical data are often not sufficient to cope with this data flow, so new, modern tools for data processing and analysis are needed, including machine learning, deep learning, Bayesian statistical methods, and others.

The Data Science in MMA program will bring together students and faculty from diverse backgrounds to enable breakthroughs in the MMA field based on deployment of modern data science tools. Students from physics and astrophysics, as well as from traditional data-science fields (statistics, computer science, electrical engineering, mathematics, and others) are encouraged to participate in



# DSMMA Program

Above all else it is **interdisciplinary and collaborative**

Physics & Astronomy, Statistics, Electrical Engineering, and  
Computer Science

**Team-taught** courses in Bayesian Astrostatistics and Machine  
Learning for Astrophysics

**Research project student teams** mentored by  
interdisciplinary teams of faculty

## Research Projects

Since 2021 I have helped mentor four projects:

- (i) a study of supernova siblings and the properties of the host galaxies
- (ii) using observed kilonova candidates to inform ejecta quantities
- (iii) cross-correlation between stochastic gravitational wave backgrounds and the cosmic microwave background
- (iv) **constraining the neutron star equation of state based on binary neutron star inspiral and post-merger gravitational wave signals.**

# Constraining the Neutron Star EoS

EoS – relationship between NS mass and radius (or, equivalently, pressure and density)

Goal is to constrain  $R_{1.6}$ , the radius of a  $1.6M_{\odot}$  NS

Current consensus:  $R_{1.6} = 12.07^{+0.98}_{-0.77}$  km (95% credible interval)

Our work:  $R_{1.6} = 11.91^{+0.80}_{-0.56}$  km (95% credible interval)

# NS EoS: Statistical Formalism and Challenges

Goal: Constrain  $R_{1.6}$

Inspiral signal provides the chirp mass

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

Post-merger signal peak frequency,  $f_{\text{peak}}$ .

Structure of Bayesian model:

$$\begin{aligned} D_i | \mathcal{M}_i, f_{\text{peak},i} &\sim p(D_{IN,i} | \mathcal{M}_i) p(D_{PM,i} | f_{\text{peak},i}) \\ \mathcal{M}_i, f_{\text{peak},i} | R_{1.6} &\sim \nu(\mathcal{M}_i, f_{\text{peak},i} | R_{1.6}) \\ R_{1.6} &\sim \nu(R_{1.6}) \end{aligned}$$

## Likelihood

$$\begin{aligned} p(D|R_{1.6}) &= \prod_{i=1}^N p((D_{\text{PM}}, D_{\text{IN}})_i | R_{1.6}) \\ &= \prod_{i=1}^N \int p(D_{\text{IN},i}, D_{\text{PM},i} | \mathcal{M}_i, f_{\text{peak},i}) \\ &\quad \times \nu(\mathcal{M}_i, f_{\text{peak},i} | R_{1.6}) df_{\text{peak},i} d\mathcal{M}_i \\ &= \prod_{i=1}^N \int p(D_{\text{IN},i} | \mathcal{M}_i) p(D_{\text{PM},i} | f_{\text{peak},i}) \\ &\quad \times \nu(\mathcal{M}_i, f_{\text{peak},i} | R_{1.6}) df_{\text{peak},i} d\mathcal{M}_i, \end{aligned}$$



## Complication

$D_i = (D_{IN,i}, D_{PM,i})$  is not available.

Instead the Bilby pipeline provides samples from

$$q(\mathcal{M}_i | D_{IN,i}) \propto p(D_{IN,i} | \mathcal{M}_i) p_0(\mathcal{M}_i)$$

and the BayesWave pipeline provides samples from

$$q(f_{\text{peak},i} | D_{PM,i}) \propto p(D_{PM,i} | f_{\text{peak},i}) p_1(f_{\text{peak},i})$$

If BayesWave does not detect a peak frequency, then samples from  $p_1(f_{\text{peak}})$  are returned.

## Likelihood

$$\begin{aligned} p(D|R_{1.6}) = & \prod_{i=1}^N \int \frac{p(\mathcal{M}_i|D_{\text{IN},i})p(D_{\text{IN},i})}{p_0(\mathcal{M}_i)} \frac{p(f_{\text{peak},i}|D_{\text{PM},i})p(D_{\text{PM},i})}{p_1(f_{\text{peak},i})} \\ & \times \nu(f_{\text{peak},i}, \mathcal{M}_i | R_{1.6}) df_{\text{peak},i} d\mathcal{M}_i \end{aligned}$$

The likelihood is not available in analytic form.

Instead we have a Monte Carlo approximation of it from the Bilby and BayesWave posterior samples.

How good is the approximation?

## Open Question

The BayesWave pipeline which is used to sample from

$$q(f_{\text{peak},i} | D_{PM,i}) \propto p(D_{PM,i} | f_{\text{peak},i})p_1(f_{\text{peak},i})$$

uses reversible jump Metropolis-Hastings to produce the samples.

RJMH is not at all well understood and has a reputation for being finicky and unreliable.

Is the BayesWave pipeline reliable?

The theory required to study this question is being developed.

# Priors

For  $R_{1.6}$  we consider two options

$$R_{1.6} \sim \text{Uniform}(9\text{km}, 15\text{km})$$

and an astrophysical prior which is the posterior reported by Huth et al (2022).

For  $\mathcal{M}$  we use priors that match values found by Abbott et al (2020) and Petrov et al (2022) which yield

$$\mathcal{M} \sim N(1.33M_{\odot}, 0.09M_{\odot})$$

## Priors

Vretinaris et al (2020) use numerical relativity simulations to derive EoS-agnostic relations between  $f_{\text{peak}}$  and  $\mathcal{M}$  of the form

$$\frac{f_{\text{peak}}}{\mathcal{M}} = g(\mathcal{M}, R_{1.6}) + \varepsilon \quad \varepsilon \sim N(0, \sigma^2)$$

but we need something of the form

$$f_{\text{peak}} = g(\mathcal{M}, R_{1.6}) + \varepsilon \quad \varepsilon \sim N(0, \sigma^2)$$

## Prior

Vretinaris et al (2020) suggests  $f_{\text{peak}}$  should be centered around

$$\mu_f = \beta_0 + \beta_1 \mathcal{M} + \beta_2 \mathcal{M}^2 + \beta_3 R_{1.6} \mathcal{M} + \beta_3 R_{1.6} \mathcal{M}^2 + \beta_3 R_{1.6}^2 \mathcal{M}$$

Formally,

$$f_{\text{peak}} \mid \text{data}, \mu_f, \lambda_f \sim N(\mu_f, \lambda_f^{-1})$$
$$\nu(\mu_f, \lambda_f) = \lambda_f^{-1/2}$$

yielding a posterior

$$q(\mu_f \mid \text{data})$$

We also consider an empirical Bayes approach to prewhitening and set

$$f_{\text{peak}} = E[\mu_f \mid \text{data}]$$

# Model Validation

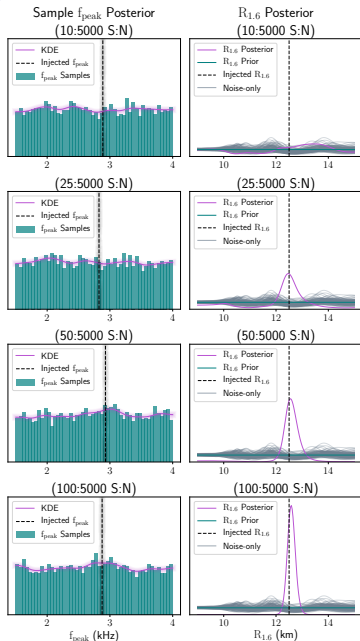
Validation: What happens if post-merger signals are just noise?

Developed a novel hypothesis testing framework based on Wasserstein distance.

Model Validation: Perform tests under ideal conditions by

- simulating BayesWave posteriors for  $f_{\text{peak}}$
- using wide range of merger masses
- using a variety of EoSs

# Ensemble of Mergers





# Model Evaluation

Simulations of future observing runs:

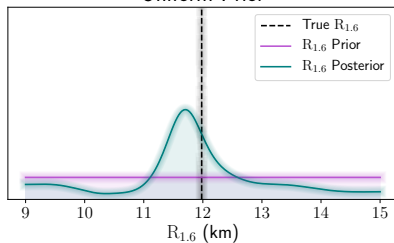
- simulate 2 one-year observing runs of LVK (O4, O5)

- Use two different EoSs

# Simulation of Future Runs

## A+ 4yr Posterior for SFHX EoS

Uniform Prior



Multimessenger Prior

