A New Tone of EM algorithm in the Universe: Analysis of MMT/Megacam Data

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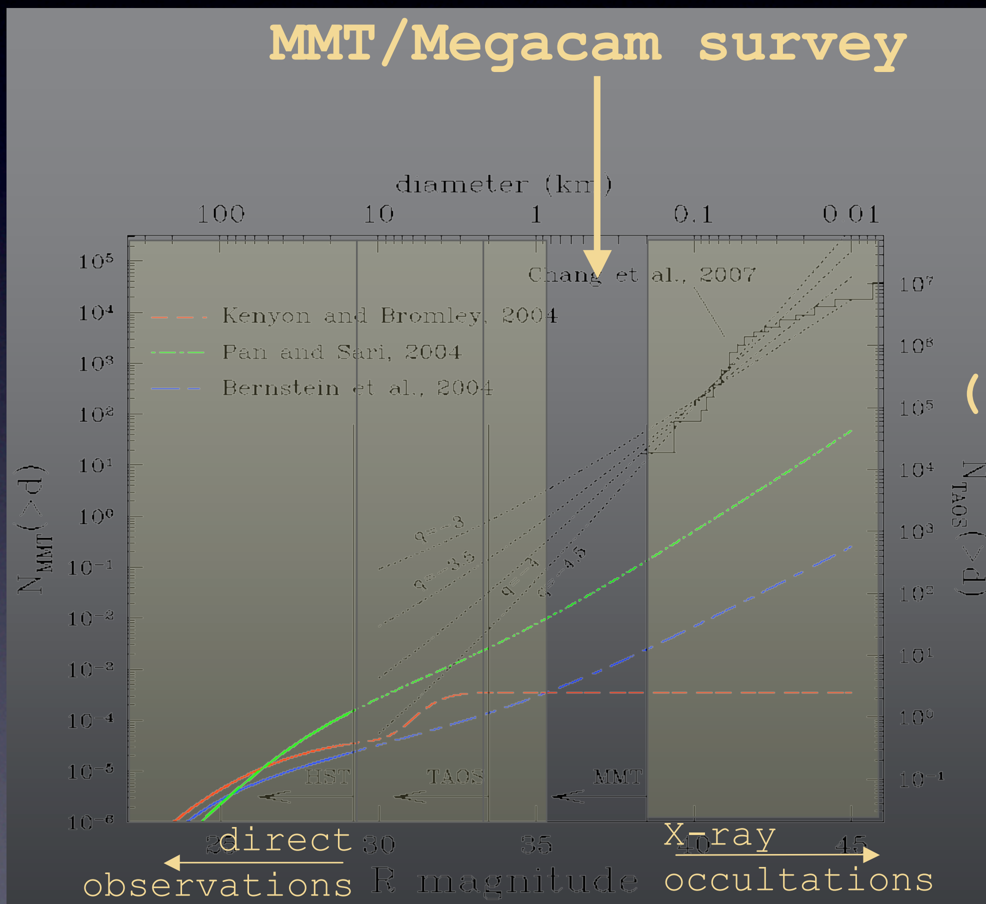
Agenda

- Backgrounds
- Models
- Some Numerical Results
- Discussion

Background

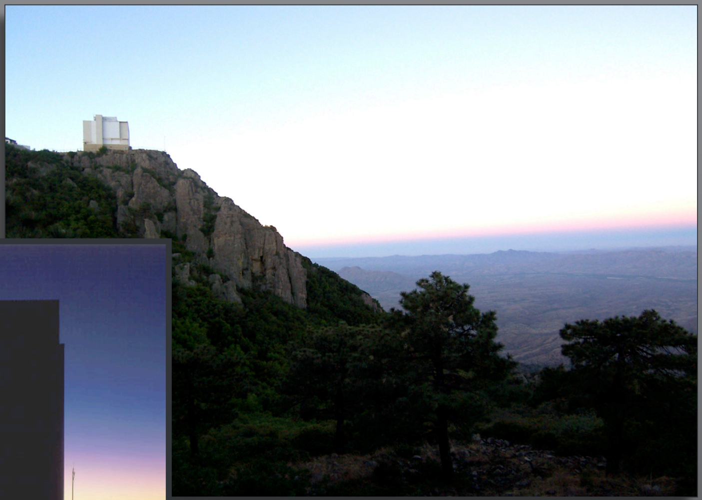
- In observing the objects in the space, there is a gap between the observable objects by direct observations and the observable objects by X-ray.
- Our Analysis of MMT/Megacam data is trying to fill the gap

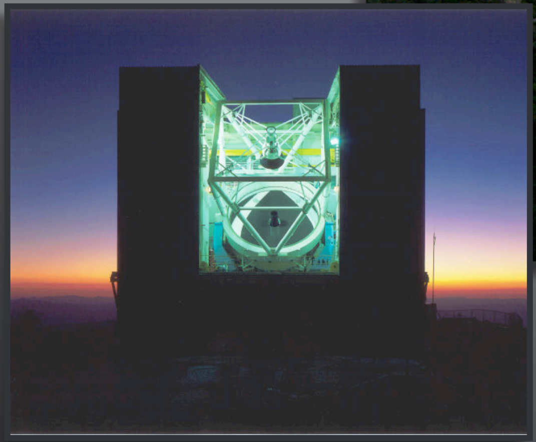
Background



target objects
200m-1km
(currently >=600m)

Background

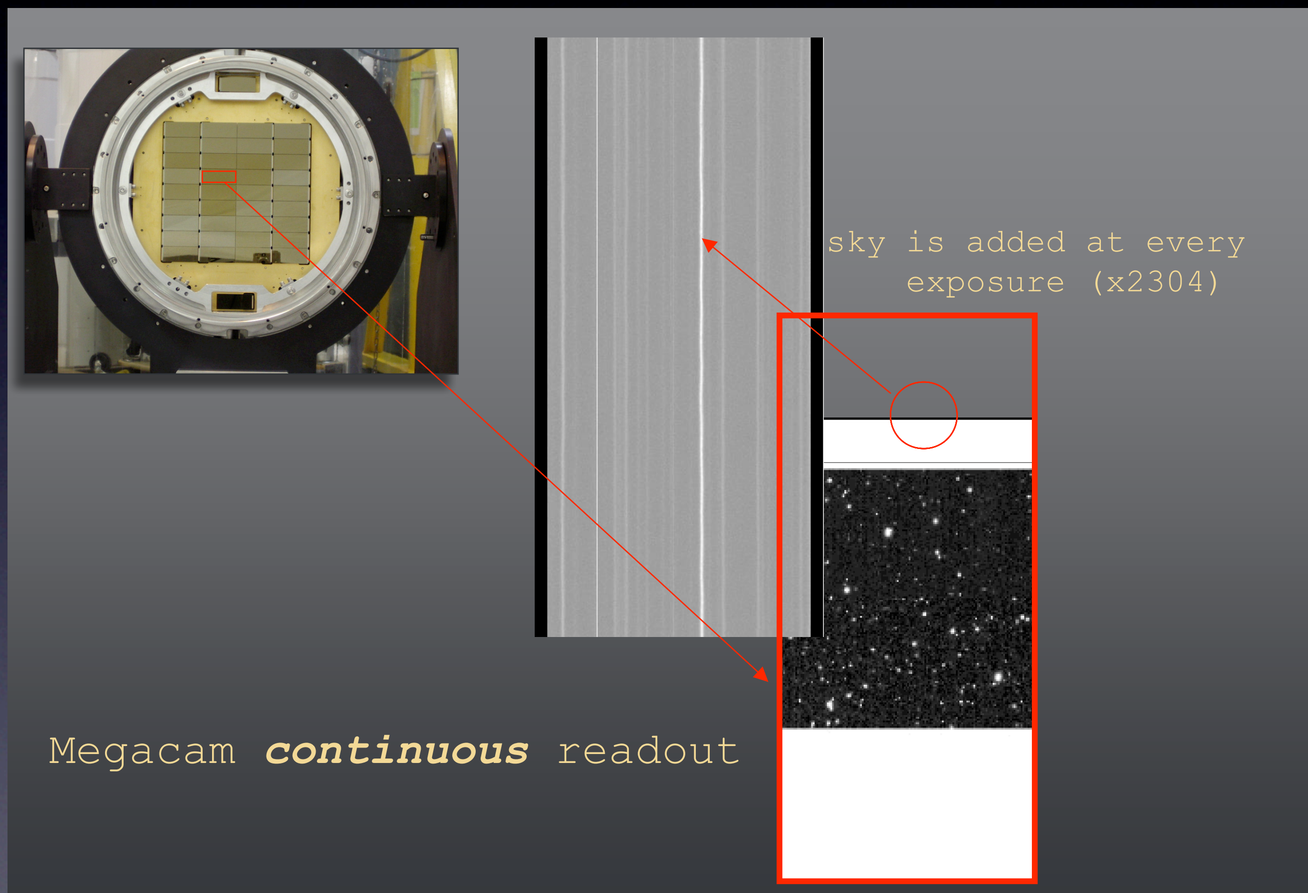




MMT is a 6.5 meter telescope on the summit of Mt. Hopkins, Arizona

Fred Lawrence Whipple Observatory

Background

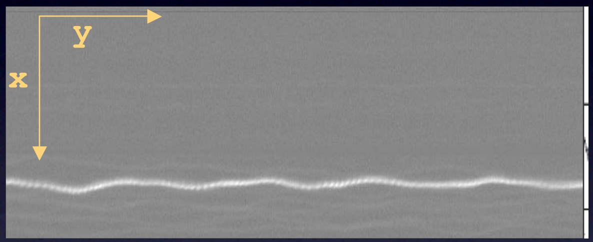

sky is added at every exposure (x2304)
Megacam ***continuous*** readout

Background

- The data from MMT/Megacam is two dimensional time series data: we have two dimensional observations of the stars and we also have a time horizon
- We will indirectly observe the targeted objects via the stars
- Want to find out the “events” when the targeted objects pass the stars

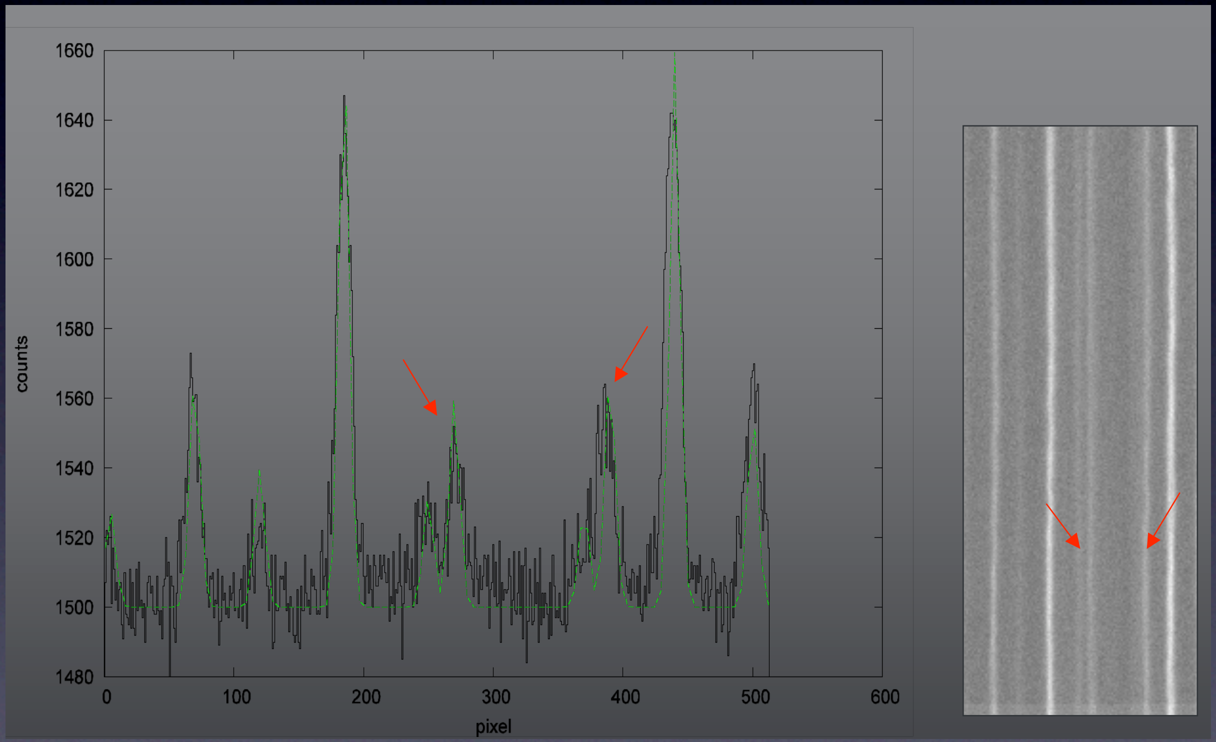
Background

- How to identify the "events"? By the fluctuation of the flux of the stars



- Need to de-convolute the effects from the stars and the background

Background



1660
1640
1620
1600
1580
1560
1540
1520
1500
1480
counts
0
100
200
300
400
500
600
pixel

Agenda

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- Models
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Models

- We will utilize the EM algorithm in the de-convolution
- Will present models with different assumptions/approaches
- Currently focus only on the de-convolution problem

Models

- We have binned data of photons, from both the background and the stars (bin size- a pixel or so)
- Notations and Setups of the question:
- Stars: $i = 1, \cdots, n$
- Bins: $j = 1, \cdots, m$
- Observed Data: $Y_{obs} = \{N_1, \cdots, N_m\}$ observed counts from each bin
- Missing Data: $Y_{mis} = \{Z_{ij}\}, i = 1, \cdots, n+1, j = 1, \cdots, m$ the photons from star i to bin j. the subscript n+1 means background

Models

- Model 1: Fixed bin counts with Poisson Backgrounds

Models

- We does not place distribution assumptions on the number of photons in each bin and we assume the background in each bin are i.i.d. poisson
- Within each bin, the number of photons from each star(background excluded) follow a multinomial distribution($\tilde{N}_j$ total photons from stars)

$$(Z_{1j}, \cdots, Z_{nj}) \sim multinomial(\tilde{N}_j, p_{1j}, \cdots, p_{nj})$$

- Normal parameterization for PSF(point spread function)

$$p_{ij} = \frac{q_i \phi(x_j; \mu_i, \sigma_i)}{\sum_{i=1}^{n} q_i \phi(x_j; \mu_i, \sigma_i)}$$

Models

- Take the background into account

$$
\begin{aligned}
Z_{n+1,j}|N_j &\sim Pois(\lambda)|Pois(\lambda) \leq N_j \\
Z_{1j}, \cdots, Z_{n,j}|N_j, Z_{n+1,j} &\sim multinomial(N_j - Z_{n+1,j}, p_{1,j}, \cdots, p_{n,j})
\end{aligned}
$$

Models

- Pros of the model: We do have the closed form solution for the updating equation:

$$
\begin{aligned}
\lambda' &= \frac{\sum_j M_j}{m} \\
q' &= \frac{\sum_j \tilde{q}_j (N_j - M_j)}{\sum_j (N_j - M_j)} \\
\mu_1' &= \frac{\sum_j \tilde{q}_j (N_j - M_j) x_j}{\sum_j \tilde{q}_j (N_j - M_j)} \\
\mu_2' &= \frac{\sum_j (1-\tilde{q}_j)(N_j - M_j) x_j}{\sum_j (1-\tilde{q}_j)(N_j - M_j)} \\
\sigma_1' &= \left(\frac{\sum_j \tilde{q}_j (N_j - M_j)(x_j - \mu_1')^2}{\sum_j \tilde{q}_j (N_j - M_j)}\right)^{1/2} \\
\sigma_2' &= \left(\frac{\sum_j (1-\tilde{q}_j)(N_j - M_j)(x_j - \mu_1')^2}{\sum_j (1-\tilde{q}_j)(N_j - M_j)}\right)^{1/2}
\end{aligned}
$$

$$
M_j = \sum_{k=0}^{N_j} \frac{k \times e^{-\lambda} \lambda^k / k!}{\sum_{l=0}^{N_j} e^{-\lambda} \lambda^l / l!}
$$

Models

- Cons of the model:
  - Tend to underestimate the background and over estimate the dispersion of the normal distribution
  - Not explicitly estimating the intensity of flux: we only estimate the proportion in each normal

Models

- Model II: Poisson bin counts with Poisson Backgrounds

Models

- We will assume that the total number of photons from a star is following poisson distribution (Esch and et al. 2004)

$$Y_j|\lambda_i, \lambda_B \sim Poisson[(\sum_i P_{ij}\lambda_i) + \lambda_B]$$

- And we will incorporate the location and dispersion of the stars through parameterization of PSF(point spread function)

Models

- Then, within the same bin(pixel), we have

$$(Z_{1j}, \cdots, Z_{nj}, Z_{n+1,j}) \sim multinomial(N_j, \frac{\lambda_1 P_{1j}}{\sum_i \lambda_i P_{ij} + \lambda_B}, \cdots, \frac{\lambda_n P_{nj}}{\sum_i \lambda_i P_{ij} + \lambda_B}, \frac{\lambda_B}{\sum_i \lambda_i P_{ij} + \lambda_B})$$

- The parameterization of the PSF:

$$p_{ij} \propto \phi(x_j; \mu_i, \sigma_i)$$

Models

- Pros:
  - Explicitly model the intensity or flux of the star through the poisson parameter
  - Better acknowledged in the research community
- Cons:
  - We do not have the closed form solution for EM iteration, which is especially undesirable for the large scale problem we have

Models

- Model 3: Hierarchical Bayes

Models

- We will assume that the total number of photons from a star is following poisson distribution (Esch and et al. 2004)
- We will use Hierarchical Bayes instead of EM algorithm to Sample the posterior distribution of intensity and the PSF

Models

- Pros: Very effective in accounting for the uncertainty of parameters
- Cons: not conjugate prior, computational concerns...

Numerical Results

- Model I:

p	μ_1	μ_2	σ_1	σ_2	λ_B
0.5(.49)	80(79.4)	30(30.0)	10(9.8)	10(11.5)	100(83.5)
0.5(.27)	80(83.2)	50(52.2)	10(8.0)	10(19.7)	100(64.4)
.3(.32)	80(79.1)	30(29.9)	10(10.4)	10(11.4)	100(75.5)

Discussion and Future work

- Need a computational effective way to de-convolute the stars with certain accuracy
- Next step: look at the time series data

Thank you and Happy Holiday!