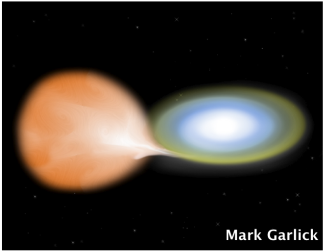
Incorporating Spectra Into Periodic Timing: Bayesian Energy Quantiles

A. Connors[1], J. Hong[2], P. Protopapas[3], V. Kashyap[4]
[1]*Eureka Scientific*, [2]*Center for Astrophysics*, [3]*Harvard-Smithsonian and School of Engineering and Applied Sciences*, [4]*Harvard-Smithsonian Center for Astrophysics.*

September 7, 2011

1. JOINT TIMING/ENERGY CHALLENGE: Poisson+Non-Parametric

Cataclysmic Variables (CVs):
Compact Binaries with white dwarf accreting from late type companion



Non–Magnetic CVs

Polars

Figure: Artist's representations of two kinds of Cataclysmic Variables (CV), containing a white-dwarf accreting matter from a main sequence companion. The X-ray apparent brightness and spectrum can change as absorbing material rotates into the line-of-sight. Left: low magnetic field; Right: high magnetic field.

Magnetic CV in Baade's Window : CXOPS J180354.3 – 300005 (1028.3 s)

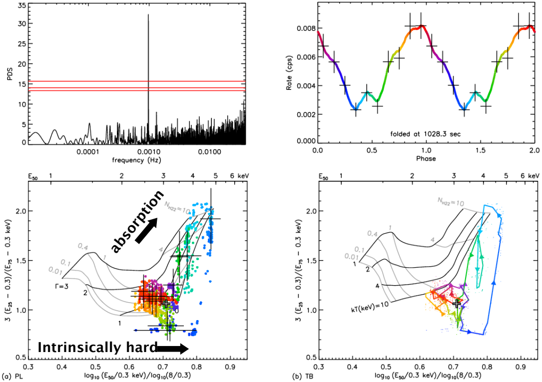
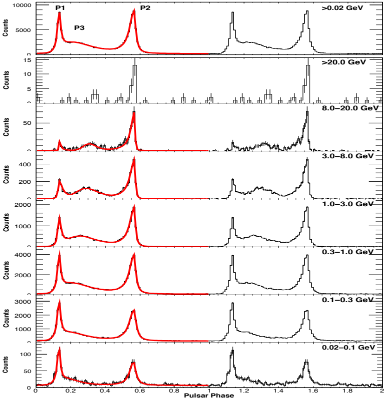


Figure: Hong et al. 2011: Joint Timing/Energy analysis of one CV. Top L: Timing analysis, power density spectrum. Bottom L: approximate energy spectrum information. (Blue = harder; red = softest). Top R: Period-folded light-curve, also displaying the changing energy spectrum.



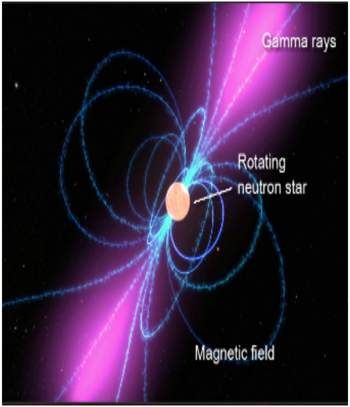


Figure: Example of an iconic $\gamma-$ray pulsar (Vela) from Abdo et al. The rates per unit phase, for different $\gamma-$ray energy bands, are illustrated on the right. L: Artist's concept of a spinning $\gamma-$ray pulsar, with this (roughly cone-shaped) high energy emission coming from high in the magnetosphere ('outer gap' models).

2. POISSON TIMING SOLUTIONS: Blocks, Various!

TRICK: For Poisson, use *Blocks of constant rate*.

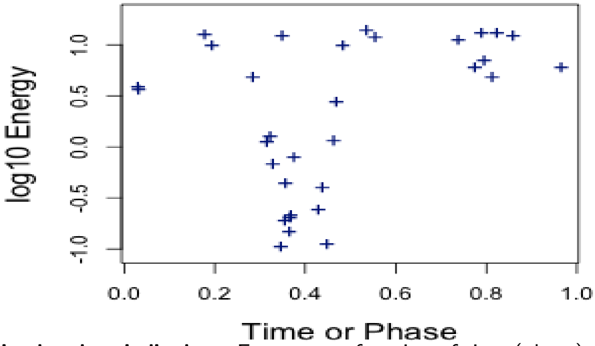


Figure: **Simulated periodic data.** Energy as a function of time (phase). It is an idealized version of the CV (CXOPS J180354.3 -300005) from Figure 2: The count-rate doubles as the exponential 'absorbtion' energy drops from 8.0 to 0.2.

2.1 Timing: Simplified Bayes Blocks (Scargle)

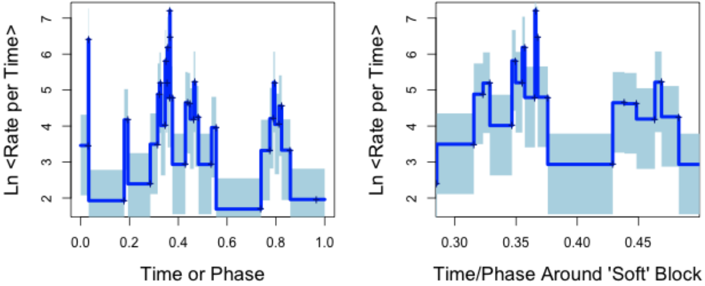


Figure: Time domain only: 'Best-fit' (blue) and 68% limits (light blue) on the 'true' rates per unit time, given our simulated data. The second panel shows a blow-up of the region around the simulated peak rates (phase 0.3-0.47).

TRICK: Independent rates in each Block; edges defined by *photon arrival times*. Uneven spacing!

* Model rate:

$$\text{Expected Cts}_i = \rho_i \Delta t_i, \; \Delta t_i = t_i - t_{i-1}$$

* Priors: ρ_i constant up to *max possible number of cts in* Δt_i.

* Results: keeps only statistically significant blocks.

2.2 Timing: Gregory and Loredo: Intriniscally Poisson Epoch-Folding

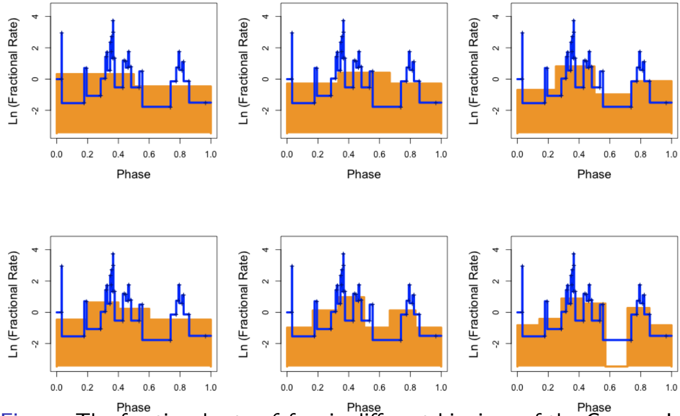


Figure: The fractional rates f for six different binnings of the Gregory-Loredo algorithm (orange). Top three panels: two, three, and four bins. Bottom three panels: five, six and seven bins. Superposed, in blue, are the model rates (see previous Fig.).

TRICK: Marginalize (Average) over m, the number of possible bins. The bins size are *even* $\frac{1}{m}$. G&L use *fractional* rates in each bin times a total rate A:
* Model rate:

$$\text{Expected Cts/bin}_i = Af_{i,m}$$

* Priors: A constant up to *max possible rate in observation*. Constant priors on the f_i, which must sum to 1.
* Result: Nice interpretation in terms of inverse of the prob (multiplicity) of the binned events by chance.

Timing: Sparse Bayes Blocks

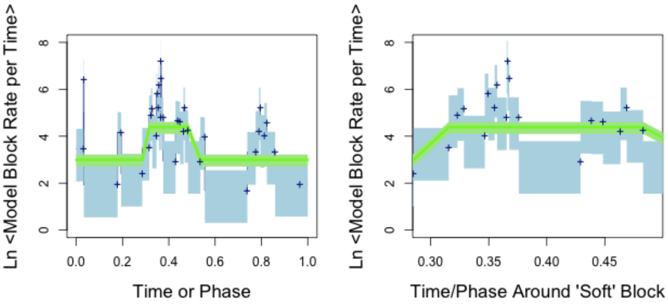


Figure: We illustrate a ‘Sparse Bayes Blocks’ periodic model. In green we show a single ‘Bayes Block’ (two change-points, two rates). (For comparison the blue shows our ‘fully saturated’ model from before.) Note that the (Sparse Bayes Blocks, green) rate at Phase =1.0 is required to match that at phase=0.0, for periodic models.

TRICK: Like Bayes Blocks but restricted to only one or two blocks. Easier for period-detection. Possible edges defined by *photon arrival times*. Uneven spacing! **This is what we will use for the 'Timing' dimension!**

* Model rate:

$$\text{Expected Cts}_i = \rho_i \Delta t_i, \;\; \Delta t_i = t_i - t_{i-1}$$

* Priors: Choice of:

- ρ_i constant up to *max possible number of cts in* Δt_i.
- ρ_i has an exponential distribution characterized by an average rate $\frac{1}{\alpha}$.

* Results: Bayesian odds ratios:

$$\mathcal{O}_{1,2,exp} = \alpha \, \frac{1}{(N_T)(N_T-4)} \sum_{CP_1,CP_2} \frac{(\alpha+\tau_T)^{N_T+1}}{(\alpha+\Delta\tau_1)^{n_1+1}\,(\alpha+\Delta\tau_2)^{n_2+1}} \frac{n_1!\;n_2!}{N_T!}; \qquad (1)$$

and

$$\mathcal{O}_{1,2,flat} = \frac{1}{(N_T)(N_T-4)} \sum_{CP_1,CP_2} \frac{\mathcal{N}_T}{\mathcal{N}_1\mathcal{N}_2} \frac{(\tau_T)^{N_T}}{(\Delta\tau_1)^{n_1}\,(\Delta\tau_2)^{n_2}} \frac{n_1!\;n_2!}{N_T!}. \qquad (2)$$

3. POISSON ENERGY-SPECTRUM SOLUTIONS: Blocks via Quantiles

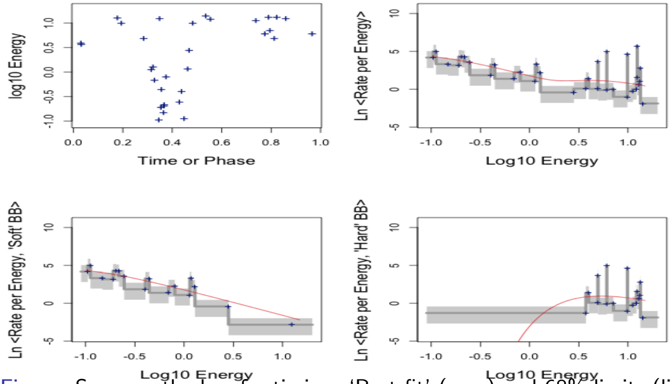


Figure: Same method as for timing: 'Best-fit' (gray) and 68% limits (light gray) on 'true' rates per unit *energy* (Upper R), Simulated data (upper L). Lower panels: Spectra for low-energy-absorption/high counts/sec and high-absorption/low counts/sec 'blocks', respectively. Red line = 'true' model.

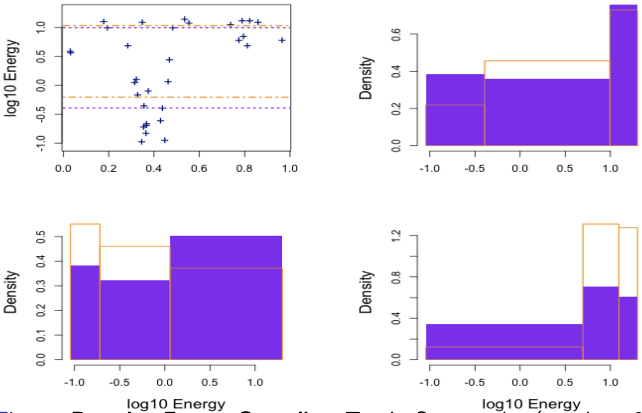


Figure: **Bayesian Energy Quantiles.** Top L: Scatter plot (purple = 25%,75% *measured* quantiles: orange = 25%,75% *true fractions* (known from simulation). Rest: histograms of *measured* (purple) and *true* (orange) *fractional rates per unit energy*; bounded by the t*measured quantiles* marking 25%. 75% of the photons. Top R: total, Lower L: softer block. Lower R: harder block.

TRICK: (Hong et al. Use *Energy Quantiles* to delineate the *Energy* blocks. Uneven binning, like Bayes Blocks; but set ahead of time, via Quantile choice (e.g. 35%, 75%, etc.)

TRICK: Model the time-rates exactly as before, with Sparse Bayes Blocks. Model the *shape of the energy spectrum* as *fraction rates per unit energy*, conditionen on the timing count-rates.

$$\text{Expected fractional rate}_i = f_i \Delta E_i, \ \Delta E_i = E_i - E_{i-1}$$

* Priors: f_i constant up to $\frac{1}{\Delta E_i}$; but with the constraint

$$\sum_i f_i \Delta E_i = 1.$$

* Results: Bayesian odds ratios:
Hence we can write a marginalized conditional likelihood ratio for just the Bayesian Energy Quantiles, conditioned on the time-rates in Bayes Blocks 1 and 2: portion:

$$\mathcal{O}\left(\{E_k\}\right)_{BEQB,12,Simple} =$$

$$\frac{(\Delta E_{0a})^{n_{0a}}}{(\Delta E_{1a})^{n_{1a}}(\Delta E_{2a})^{n_{2a}}} \times \frac{(\Delta E_{0b})^{n_{0b}}}{(\Delta E_{1b})^{n_{1b}}(\Delta E_{2b})^{n_{2b}}} \ldots \times \frac{(\Delta E_{0m})^{n_{0m}}}{(\Delta E_{1m})^{n_{1m}}(\Delta E_{2m})^{n_{2m}}} \times$$

$$\frac{n_{1a}!\ n_{2a}!}{n_{0a}!} \times \frac{n_{1b}!\ n_{2b}!}{n_{0b}!} \ldots \times \frac{n_{1m}!\ n_{2m}!}{n_{0m}!} \times \frac{N_T!}{n_1!\ n_2!} \tag{3}$$

This simple form has many advantages. It is easy to code. It is straightforward to generalize to more timing 'blocks', including the binned G&L algorithm; as well as more energy quantile 'blocks'. It is also straightforward to generalize to *spatial* analysis based on blocks!
Preliminary results on simulated and real CV data look promising!

4. PRELIMINARY RESULTS

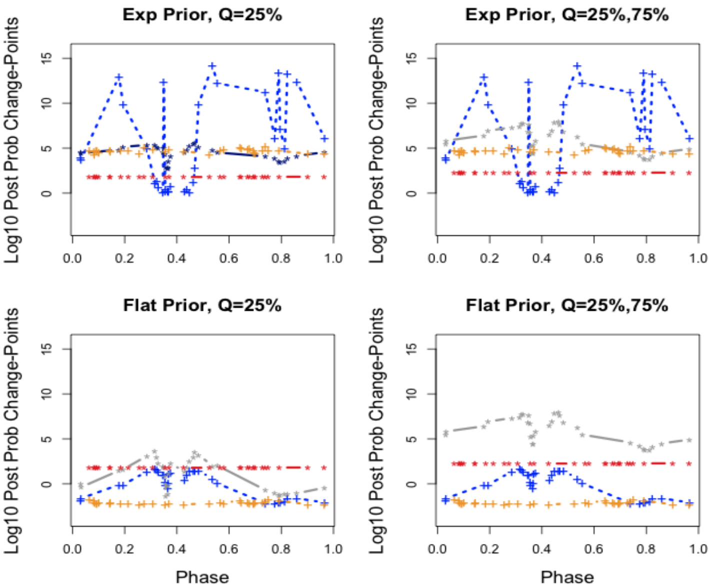

Exp Prior, Q=25%
Exp Prior, Q=25%,75%
Flat Prior, Q=25%
Flat Prior, Q=25%,75%
Log10 Post Prob Change-Points
0
5
10
15
0.0
0.2
0.4
0.6
0.8
1.0
Phase

Figure: **Results.** $\log_{10}$ Odds for the positions of two change-points, on our simulated CV data (gray and blue); and simulated null data (32 events, flat time-rate, constant spectrum; red and orange). Simulated CV: blue = $\log_{10}$ Odds for time-rates only; dark gray = Total $\log_{10}$ Odds (includes Bayesian Energy Quantile analysis at $F_a = 0.25$). Null: orange = $\log_{10}$ Odds for time-rates only; red = Total $\log_{10}$ Odds (includes Bayesian Energy Quantile analysis) at $F_a = 0.25$.

SIMULATED CV and NULL:

Source	Log10 Odds Exp Prior of: rate only	w/Q 25%	w/Q 25%,75%	Log10 Odds Flat Prior rate only	w/Q 25%	w/Q 25%,75%
NULL	-1.04	-0.686	-0.578	-2.09	-1.73	-1.63
Sim CV	0.482	4.80	2.50	-0.0956	4.13	1.93

REAL CV W/ PERIOD 4731 S:

Period (s)	Log10 Odds Exp Prior of: rate only	w/Q 25%	w/Q 25%,75%	Log10 Odds Flat Prior rate only	w/Q 25%	w/Q 25%,75%
1020.	-2.72	-1.27	-1.26	-3.52	-2.07	-2.01
1573.	-2.03	-0.660	-0.340	-2.81	-1.43	-1.12
3000.	-3.01	-2.01	-1.83	-3.76	-2.80	-2.50
4731.	6.34	6.95	7.54	5.81	6.43	7.01
6200.	-2.99	-2.37	-2.18	-3.73	-3.10	-2.85
9461.	0.319	0.743	0.913	-0.340	0.0346	0.245

NOTE: $\log_{10}$ Odds of ~ 7 means: Null hypothesis is 10^{-7} *less likely* than our main hypothesis – so this is many, many 'σ's!

REFERENCES

Hong et al. (2009, 2011)
Abdo Et Al Fermi Collab VelaPulsar Fig2

`arXiv1002_4050v1`

Hong, Schlegel & Grindlay, 2004
Huijse, Zegers & Protopapas (2011) on co-entropy
Scargle on Bayes Blocks
Gregory and Loredo
Connors; Connors and Carraminana Tutorials in SCMA IV.