Two Statistical Problems in X-ray Astronomy

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Outline

Introduction

- Recent projects have focused on two areas:
  - Analysis of faint (low-count) x-ray data with Bayesian models
  - Analysis of events in time series
- Each has presented a unique set of challenges

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- This frequently leads to a sample containing many faint, undetected sources.
- We want to combine information from these undetected sources to make inferences about our selected sample.

Current method: stacking

- Based on background subtraction
- For source i, observe $c_{s,i}$ counts in source aperture and $c_{b,i}$ counts in background aperture.
- Calculate net counts as $c_{n,i} = c_{s,i} - \frac{A_{s,i}}{A_{b,i}} c_{b,i}$, where $A_{s,i}$ and $A_{b,i}$ are the effective areas for the source and background regions (taking into account exposures), respectively.

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- Calculate stacked flux as $\bar{f}_x = \frac{\overline{\mathrm{ECF}}}{\sum_i A_{s,i}} \sum_i c_{n,i}$, where $\overline{\mathrm{ECF}}$ is the mean energy conversion factor.
- Calculate stacked luminosity as $\bar{L}_x = \frac{1}{N} \sum_i \mathrm{LCF}_i c_{n,i}$, where LCF_i is the luminosity conversion factor for source i
  - $\mathrm{LCF}_i = \frac{4\pi d_{\ell,i}^2 \mathrm{ECF}_i \times A_{corr,i} \times K_{corr,i}}{A_{s,i}}$

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- No clean measure of uncertainties on luminosities.
- Solution: model data as Poisson

A hierarchical Bayesian model for "stacking"

Observation Model

- For source i, we assume that $c_{n,i} \sim \text{Pois}(\lambda_{n,i})$
- Also assume $c_{b,i} \sim \text{Pois}(\lambda_{b,i} \frac{A_{b,i}}{A_{s,i}})$
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- Using noninformative priors on hyperparameters (Jefferys)

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  - Relatively plausible for spectroscopic; not as much for photometric
- Assuming the spectra of sources are know & identical
  - Typically assume power law with photon index ≈ 1.7
- Attempting to make inferences only on selected sample, for now; not dealing with selection effects, etc.

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  - MH step here is very efficient; using Haley's method to identify posterior modes in parallel and tune proposal distribution.
- From posterior simulations, can retain posterior mean & standard deviation of each source flux (and luminosity, if available) in addition to hyperparameter samples.
- This provides a great deal of information that is not available with conventional stacking in addition to estimates of sample properties with uncertainties.

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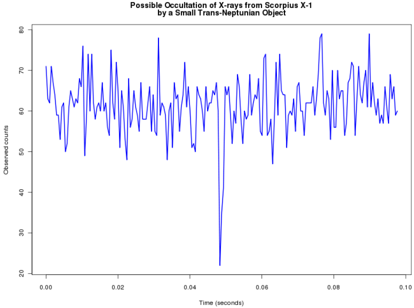
  - Incorporation of photometric redshift uncertainties

Testing time symmetry for astronomical events



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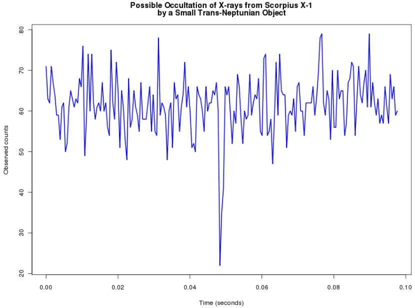

- We have a set of x-ray light curves like the above, each of which is believed to contain an event (in this case, an occultation).
- Interested in testing if the event (a dimming, in this case) is time-symmetric.

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  - Question of how much structure to place on shape of event.
  - Taking maximum over possible centers of event for less structured approach $\Rightarrow$ complex distribution of test statistic.
- With Poisson data, we really need a structured model.

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where $\lim_{t \to \infty} g(t; \tau, \theta) = \lim_{t \to -\infty} g(t; \tau, \theta) = 0$ and $\sup_{\mathbb{R}} g(t; \tau, \theta) = g(\tau; \tau, \theta) = 1$

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- Thus, c characterizes our baseline source intensity, α characterizes the extent of the deviation from this baseline during the event, and $g(t; \tau, \theta)$ characterizes the shape of the event itself.

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- This approach generalizes easily to the high count regime with only minor modifications.

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$$g(t;\tau,h_1,h_2,k_1,k_2)=\frac{1+e^{\frac{-h_t}{k_t}}}{1+e^{\frac{|t-\tau|-h_t}{k_t}}}$$

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- Because data is non-Gaussian, still need to simulate under null hypothesis to obtain actual distribution of test statistic (cannot necessarily rely on χ^2 approximation).

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- This approach parallelizes extremely well, making it ideal for use in a cluster environment (such as Odyssey).