# Astro-Statistics: What is it good for?

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# Astro-Statistics

DvD: "Statistics applied to Astronomy"

Astro perspective: develop algorithms to infer astronomical truth Keep astronomers from fooling themselves

Astronomical data are generally cleaner – there is less uncorrectable bias Loads of BIG data Some unique circumstances like well-defined calibration One-shot experiments that require Bayesian analyses High-energy datasets doubly simpler, being recordings of a Poisson point process





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# High-Energy Astrophysics

X-rays and Gamma-rays  $< 10^{-6}$  cm or  $> 2x10^{16}$  Hz Not visible from the ground - Space-based observations



# Sources of High-Energy Radiation

- Stellar Coronae
- Supernova remnants
- Galactic outflows
- Clusters of galaxies
- Compact objects: neutron stars, accreting black holes, supermassive black holes
- Relativistic jets
- GRBs
- etc...











## Data in High-Energy Astrophysics

- X-ray and  $\gamma$ -ray data count photons => Poisson in nature
- Complex physics and data collection
- Data may exhibit Spectral, Temporal and Spatial variations



Crab Nebula - variations during 6 month of snap-shot observations with Chandra X-ray Observatory



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## **Data Collection in Space**

### Chandra X-ray Observatory





### **Telescope + Detectors**

Measurement Process Inefficient data collection Process Instrument characteristics Instrument Calibration Interstellar Medium

Loss of signal but also imprints information Astronomical Object

### Physics



# **Data Collection**

Energy Spectra 1D

- Data are recorded for each arriving photon:
  - the (2-dimensional) location sky coordinates
  - the photon energy
  - the arrival time
- All variables are discrete
  - High resolution -> finer discretization,
  - e.g., 4096 x 4096 spatial or up to 16384 spectral bins
- Table with photon counts for:
  - Spectral analysis 1D
  - Spatial analysis 2D
  - Timing analysis 1D







Chandra X-ray Image

## Instrumental Effects: Recording inefficiency

- Image:
  - exposure map
  - "sensitivity to photons per area"
- Spectrum:
  - effective area (ARF)
  - "sensitivity to photons per energy"







## Instrumental Effects: Blurring

### Image

- point source observed size depends on the source location on the detector
- "blurring" is described by a point spread function (PSF)

## Spectrum

- photon energy is "blurred"
- probability of detecting photon at given energy in given detector channel is described by a redistribution matrix (RMF)



**PSF** Simulated





## Instrumental Effects: Blurring

### PSF Simulated Images

### Image

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# Challenges

- Sparse, locally saturating, Poisson data
- Instrumental effects
- Source Detection in Deep Images
- Irregular extended structures
- Source boundaries
- Complex physical models
- Non-periodic, stochastic variability







#### Faculty/Researchers

#### Statisticians

David van Dyk, Imperial College London Nathan Stein, University of Pennsylvania Paul Baines, University of California, Davis Thomas Lee, University of California, Davis Xiao-Li Meng, Harvard Yaming Yu, University of California, Irvine Astronomers

Andreas Zezas, Crete

Aneta Siemiginowska, Harvard-Smithsonian Center for As Kaisey Mandel, Harvard-Smithsonian Center for Astrophy Vinay Kashyap, Harvard-Smithsonian Center for Astrophy

#### Associates

Alex Young, NASA-GSFC Pavlos Protopapas, Harvard Peter Freeman, Carnegie Mellon Taeyoung Park, Yonsei

#### Former Faculty / Researchers / Associates

Alanna Connors Eric Kolatcyk, Boston University James Chiang, Stanford Rima Izem

#### Former Post Docs

Brandon Kelly (Hubble Fellow), UCSB Hyunsook Lee, Korea Institute of S&T Evaluation and Planning

#### International CHASC Astro-Statistics Collaboration

This page lists resources of specific interest to astronomers. For detailed descriptions and reports of C-BAS/ICHASC activities, see <a href="http://www2.imperial.ac.uk/~dvandyk/astrostat.php">www2.imperial.ac.uk/~dvandyk/astrostat.php</a>

Software | Activities | Bibliography | Astro jargon | Stat jargon | People | Mailing-List | Internal

ostat-announce GoogleGroup | GoogleCalendar | AstroStat Slog Archive



#### PhD Students

Dan Cervone (Harvard) David Jones (Harvard) David Stenning (UC Irvine) Hyungsuk Tak (Harvard) Irina Udaltsova (UC Davis) Lazhi Wang (Harvard) Minjie Fan (UC Davis) Qi Gao (UC Davis) Shijing Si (Imperial) Vasileios Stampoulis (Imperial) Xiyun Jiao (Imperial)

#### CHASC was founded in 1997

#### Former Students

CJ Zijin Shen (Harvard 2000), Head of Options Trading, Jump Trading, LLC Chris Hans (Harvard AB 2001), Associate Professor, Ohio State University Rostislav S. Protassov (Harvard 2003) Director, Citigroup David Esch (Harvard 2003), Director of Research, New Frontier Advisors Hosung Kang (Harvard 2005), Quantitative Analyst, Graham Capital Management Yaming Yu (Harvard 2005), Associate Professor, Univ of California, Irvine Taeyoung Park (Harvard 2006), Associate Professor, Yonsei Univ., Korea Alan Burton Lenarcic (Harvard 2009), Manhattan Securities and Exchange Commi-Paul David Baines (Harvard 2010), Assistant Professor, Univ of California, Davis Xianchao Xie (Harvard 2011), Two Sigma Investments Li Zhu (Harvard 2012), Getco LLC Jingchen Liu (Harvard 2008), Assistant Professor, Columbia Victoria Liublinska (Harvard 2013), College Fellow in Statistics, Harvard Nathan Stein (Harvard 2013), Visiting Assistant Professor, University of Pennsylva Alex Blocker (Harvard 2013), Google Jin Xu (UC Irvine 2014), Adobe Shandong Zhao\* (UC Irvine 2014), Apple Raymond Wong\* (UC Davis 2014), Assistant Professor, Iowa State University

### BLoCXS

CJ Shen / Chris Hans / Rostislav Protassov / Yaming Yu / Taeyoung Park / Hyunsook Lee / Jin Xu / Shandong Min



van Dyk, D. A., Connors, A., Kashyap, V. L., Siemiginowska, A. (2001) Analysis of Energy Spectra with Low Photon Counts via Bayesian Posterior Simulation. The Astrophysical Journal, 548, 224-243.



## **Highly Structured Statistical Models**



van Dyk et al. 2001



Complex data collection needs to be included in the statistical model:



Observed counts are modeled as independent Poisson variables with  $\,\lambda$  mean

### BLOCXS / ppp Rostislav Protassov / Yaming Yu / Taeyoung Park

Protassov LRT

- plot of LRT distributions line detection



F-test was being commonly misused in astro analyses because of a lack of appreciation of the asymptotic conditions under which it was valid.

posterior predictive p-values for LRTs

Protassov+ 2002, became our most famous paper has been cited 301 times

Protassov, R., van Dyk, D. A., Connors, A., Kashyap, V. L. and Siemiginowska, A. (2002). *Statistics: Handle with Care, Detecting Multiple Model Components with the Likelihood Ratio Test.* ApJ, 571, 545-559.

Park, T. van Dyk, Siemiginowska, A. (2008) -Searching for Narrow Emission Lines in X-ray Spectra: Computation and Methods, ApJ. 688, 807

## LRT

- » Assumptions of the Likelihood Ratio Test statistics:
  - The null hypothesis must be a special case of the alternative
  - The parameter space of the null must be interior of the alternative parameter space.
- » The second assumption fails when testing for a spectral emission line:
  - When there is no line, the line intensity is zero, it may not be negative.
  - The line locations and width of the line do not exist when there is no line. They have no values.

LRT

### **IMPORTANT!** We do not know the true distribution of the test statistics.



» Results of three tests compared to the nominal  $\chi^2$  distribution

Protassov et al. (2002)

pyBLoCXS / Calibration Yaming Yu / Taeyoung Park / Hyunsook Lee / Jin Xu

## Foundations of Astronomical inference: Measurement Significance Calibration

Calibration is not perfect, it has known statistical and systematic errors, and unknown errors that are only guessed at.

Drake, J.J., et al. 2006, "Monte Carlo processes for including Chandra instrument response uncertainties in parameter estimation studies", SPIE Proc. 6270, 49 Kashyap, V.L., et al. 2008, "How to handle calibration uncertainties in high-energy astrophysics", SPIE Proc. 7016, 21 Lee, H., et al. 2011, "Accounting for Calibration Uncertainties in X-ray Analysis: Effective Areas in Spectral Fitting", ApJ, 731, 126 Xu, J., et al. 2014, "A Fully Bayesian Method for Jointly Fitting Instrumental Calibration and X-ray Spectral Models", ApJ, in press



coldb.v9900.v9901.v9989.v9990

Yaming Yu / Taeyoung Park / Hyunsook Lee / Jin Xu / Shandong Min

# fitting to simulated data $f(\varepsilon; \theta) = \theta_3 \varepsilon^{-\theta_1} e^{-\theta_2 \sigma(\varepsilon)}$



Yaming Yu / Taeyoung Park / Hyunsook Lee / Jin Xu / Shandong Min

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 $p(\theta|D,A_0)$ 

Yaming Yu / Taeyoung Park / Hyunsook Lee / Jin Xu / Shandong Min

# fitting to simulated data $f(\varepsilon; \theta) = \theta_3 \varepsilon^{-\theta_1} e^{-\theta_2 \sigma(\varepsilon)}$



## $p(\theta|D,A_0)$ $p(\theta|D,A_i)$

Yaming Yu / Taeyoung Park / Hyunsook Lee / Jin Xu / Shandong Min

# fitting to simulated data $f(\varepsilon; \theta) = \theta_3 \varepsilon^{-\theta_1} e^{-\theta_2 \sigma(\varepsilon)}$



 $p(A) p(\theta | D, A)$ 

 $p(\theta | D, A_0)$ 

 $p(\theta | D, A_i)$ 

Yaming Yu / Taeyoung Park / Hyunsook Lee / Jin Xu / Shandong Min

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 $p(\theta | D, A_i)$ 

 $p(A(\theta'), \theta|D)$ 



### Bayesian Multi-scale reconstruction of low-counts images





Esch, D.N., Connors, A., Karovska, M., & van Dyk, D.A., 2004, *An Image Restoration Technique with Error Estimates* ApJ, 610

### LIRA Nathan Stein / Katy McKeough





## Significance of irregular structure



McKeough +2014, Stein+ 2014

### Hardness Ratios Chris Hans / Yue Wu / Taeyoung Park

### Simplest measure of shape of a spectrum. Use counts in passbands, $C_i \sim Pois(a_i\lambda_i + b_i)$ , to compute $p(R = \lambda_1/\lambda_2 | C_i)$ $p(HR = (\lambda_1 - \lambda_2)/(\lambda_1 + \lambda_2) | C_i)$ $p(C = \log(R) | C_i)$



Park, T., et al. 2006, "Bayesian Estimation of Hardness Ratios: Modeling and Computations", ApJ, 652, 610

Hardness Ratios Taeyoung Park

- ◆ BEHR is used in the Chandra Source Catalog
- Next step: pySALC infer spectral model parameters directly





- Bounds: Confidence/Credible range estimates on parameters
- Limits: You make an observation, find nothing, and ask at what brightness would the source have been detected, and conclude it must be dimmer than that.



Kashyap, V.L., et al. 2010, "On Computing Upper Limits to Source Intensities", ApJ, 719, 900

### **Differential Emission Measure**

Hosung Kang / Viktoria Liublinska / Nathan Stein

### $f_{\text{ul};\lambda} = \int d\log T G_{\text{ul};\lambda}(T,n_e) A_Z n_e^2 dV/d\log T$

Kashyap, V. & Drake, J.J., 1998, "Markov-Chain Monte Carlo Reconstruction of Emission Measure Distributions: Application to Solar Extreme-Ultraviolet Spectra", ApJ, 503, 450
Kang, H., et al., 2003, "A Response Matrix Approach to the Reconstruction of Differential Emission Measure", AAS/ SPD 34, 02.01, BAAS 35, p807
Kang, H., et al. 2004, "Reconstructing Stellar DEMs from X-ray Spectra", AAS/HEAD 8.0501
Kang, H, et al. 2005, "Incorporating Atomic Data Errors in Stellar DEM Reconstruction", in X-Ray Diagnostics of Astrophysical PlasmasL Theory, Experiment, and Observation, AIP Conf. Proc., v774, p373

### **Differential Emission Measure**

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$$f_{\mathrm{ul};\lambda} = \int d\log T \, G_{\mathrm{ul};\lambda}(T,n_e) \, A_Z \, n_e^2 \, dV/d\log T \, \mathrm{DEM}(\mathsf{T})$$

Kashyap, V. & Drake, J.J., 1998, "Markov-Chain Monte Carlo Reconstruction of Emission Measure Distributions: Application to Solar Extreme-Ultraviolet Spectra", ApJ, 503, 450
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Solar DEMs Nathan Stein

Unlike stellar gratings data, Solar data have high spatial resolution, low spectral and temporal resolutions. Also, high data rate. Large images in multiple filters at ≈12 sec cadence.

Bypass DEM generation and compute thermal segmentation directly from the data.

Stein, N.M., et al. 2012, "H-means Image Segmentation to Identify Solar Thermal Features", In IEEE International Conference on Image Processing (ICIP). (student paper award finalist)
Stein, N.M., 2014, "Detecting Thermal Features in Massive Streams of Solar Images", in Big Data in Astro Statistics, Section on Statistical Learning and Data Mining, JSM

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SDO/AIA- 335 /

SDO/AIA- 211 1

SDO/AIA- 193 1

SDO/AIA- 171 20101002\_233313





## SDO/AIA 2010 Oct 2 05:57



## SDO/AIA 2010 Oct 2 18:43

## Sunspots: Cycle

Yaming Yu / David Stenning



Yu, Y., et al. 2012, "A Bayesian Analysis of the Correlations Among Sunspot Cycles", Solar Physics, 281, 847 Stenning, D., et al. 2014, "A Bayesian Analysis of the Solar Cycle Using Multiple Proxy Variables", Current Trends in Bayesian Methodology with Applications, Editors: S. Upadhyay, D.K. Dey, U. Singh and A. Loganathan, Chapman & Hall/CRC Press, in press



David Stenning



Stenning, D., et al. 2012, "Morphological Image Analysis and its Application to Sunspot Classification", Statistical Challenges in Modern Astronomy V (Editors: G.J. Babu and E.D. Feigelson), Springer Verlag, New York, 2012 Stenning, D., et al., 2013, "Morphological feature extraction for statistical learning with applications to solar image data", in Statistical Analysis and Data Mining, DOI: 10.1002/sam.11200



David Stenning



Extract scatter for both polarities, relative strength, curvature of separator, and penumbral polarity overlaps

Stenning, D., et al. 2012, "Morphological Image Analysis and its Application to Sunspot Classification", Statistical Challenges in Modern Astronomy V (Editors: G.J. Babu and E.D. Feigelson), Springer Verlag, New York, 2012 Stenning, D., et al., 2013, "Morphological feature extraction for statistical learning with applications to solar image data", in Statistical Analysis and Data Mining, DOI: 10.1002/sam.11200



David Stenning



Extract scatter for both polarities, relative strength, curvature of separator, and penumbral polarity overlaps **Table 1.** Confusion matrix of the random forest predictions on out-of-bag data.

		Manual classification			
		α	β	βγ	βγδ
Automatic	α	25	1	0	0
classification	β	2	63	5	0
	βγ	0	1	11	1
	βγδ	0	0	2	8

Stenning, D., et al. 2012, "Morphological Image Analysis and its Application to Sunspot Classification", Statistical Challenges in Modern Astronomy V (Editors: G.J. Babu and E.D. Feigelson), Springer Verlag, New York, 2012 Stenning, D., et al., 2013, "Morphological feature extraction for statistical learning with applications to solar image data", in Statistical Analysis and Data Mining, DOI: 10.1002/sam.11200

### Color-Magnitude Diagrams Paul Baines / Nathan Stein

**Magnitude** = -log(brightness) (relative to a standard) **Color** = difference in Magnitudes at different wavelengths



http://sciencevault.net/ibphysics/astrophysics/pics/hrdiagram1.gif

### Color-Magnitude Diagrams

Paul Baines / Nathan Stein

NGC104 -- 47TUC CMD



DeGennaro, S., et al. 2008, "Inverting Color-Magnitude Diagrams to Access Precise Star Cluster Parameters: A New White Dwarf Age for the Hyades", ApJ, 696, 12 van Dyk, D.A., et al. 2009, "Statistical Analysis of Stellar Evolution", Annals of Applied Statistics, 3, 117 Jeffery, E.J., et al., 2011, "The White Dwarf Age of NGC 2477", ApJ, 730, 35 Stein, N.M., et al. 2013, "Combining Computer Models to Account for Mass Loss in Stellar Evolution", Statistical Analysis and Data Mining, 6, 34

### logN-logS

Nondas Sourlas / Paul Baines / Irina Udaltsova / Raymond Wong



Kong et al. 2003, via A. Zezas

 $\frac{dN}{dS} = KS^{-\alpha - 1}$  $N(>S) = KS^{-\alpha}$  $N(>S) = \begin{cases} K_1 S^{-\alpha_1} & S \ge S_b \\ K_2 S^{-\alpha_2} & S_0 < S < S_b \end{cases}$ 

Wong, R., et al. 2014, "Automatic Estimation of Flux Distributions of Astrophysical Source Populations", Annals of Applied Statistics, in press Udaltsova, I., et al., 2011, "log(N)-log(S): A Measuring Stick for the Universe", SCMA V Udaltsova, I., et al., 2014a,b, in preparation

## logN-logS

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$$\frac{dN}{dS} = KS^{-\alpha-1}$$

$$N(>S) = KS^{-\alpha}$$

$$N(>S) = \begin{cases} K_1S^{-\alpha_1} & S \ge S_b \\ K_2S^{-\alpha_2} & S_0 < S < S_b \end{cases}$$

Some recent capabilities:

- ★ ability to determine number of segments
- hierarchical Bayesian modeling
- ✦ allow for detection efficiency
- multiple-Pareto models
- posterior predictive p-value checks
- ♦ base sensitivity limit

Wong, R., et al. 2014, "Automatic Estimation of Flux Distributions of Astrophysical Source Populations", Annals of Applied Statistics, in press Udaltsova, I., et al., 2011, "log(N)-log(S): A Measuring Stick for the Universe", SCMA V Udaltsova, I., et al., 2014a,b, in preparation

# Luminosity Functions



Blocker et al. 2009, *X-ray Stacking for the Analysis of Faint Sources: A Bayesian Alternative,* Proceedings of the conference held 22-25 September, 2009 in Boston, Chandra





Wang, L., et al. 2014, "Catalog-based X-ray Population Modeling", in preparation





J independently derived catalogs of sources (from optical/IR/radio observations) *Question*: Can we bypass X-ray detection process to infer properties of the underlying luminosity function? Are detectable X-ray sources representative of the whole sample?

Wang, L., et al. 2014, "Catalog-based X-ray Population Modeling", in preparation

### Luminosity Functions Lazhi Wang



Figure 9: The histograms of the posterior draws of  $\mu$  (left),  $\theta$  (middle) and  $\pi_d$  (right). The dataset includes only the 649 sources within 6-arcmin from the center of the field. The red solid lines are the posterior mode estimators and the blue dash lines show the lower and upper bound of the 95% HPDI of the parameters.

Wang, L., et al. 2014, "Catalog-based X-ray Population Modeling", in preparation

### Spatio-Spectral Disentangling David Jones



Orion Nebula Cluster ObsID 1522

Jones, D., et al. 2014, "Disentangling Overlapping Astronomical Sources Using Spatial and Spectral Information", in preparation



Jones, D., et al. 2014, "Disentangling Overlapping Astronomical Sources Using Spatial and Spectral Information", in preparation

### Spectro-temporal Partitioning Raymond Wong

With high resolution available in both spectral and temporal regimes, compute a 2D Bayesian Blocks like partitioning of the data. Segment data at points where both intensity and spectral shape change significantly. Fit lines+continuum model to spectra in small time bins, compute likelihood that fitted spectra are different, group time bins with similar spectra.



Wong, R., et al. 2014, "Detecting Abrupt Changes in the Spectra of High-Energy Astrophysical Sources", in preparation

## Event Detection in Time Series

Alex Blocker



Blocker, A., and Protopapas, P., 2013, *"Semi-parametric Robust Event Detection for Massive Time-Domain Databases"*, SCMA V, Springer-Verlag, p177; http://arxiv.org/abs/1301.3027

## Event Detection in Time Series

Alex Blocker



MACHO 104.20121.1692.0



Blocker, A., and Protopapas, P., 2013, "Semi-parametric Robust Event Detection for Massive Time-Domain Databases", SCMA V, Springer-Verlag, p177; http://arxiv.org/abs/1301.3027

# Event Detection in Time Series



MACHO 104.20121.1692.0



Multi-stage analysis, first with simpler tools, then with model checking and uncertainty assessment.

Separate medium-frequency "candidates" from low-frequency trends and high-frequency noise via wavelet-based statistical fitting.

Blocker, A., and Protopapas, P., 2013, "Semi-parametric Robust Event Detection for Massive Time-Domain Databases", SCMA V, Springer-Verlag, p177; http://arxiv.org/abs/1301.3027

### 1.3691.19 B Cepheid

### 1.3570.1180 B RR

1.3931.98 B EB

### 2.5267.1362 B LPV









### Challenges Banff Higgs Paul Edlefsen

*Introduction.* This article addresses the problem of estimating **a Poisson rate in the presence of additive and multiplicative noise**, when additional measurements provide data for estimating the nuisance parameters. The problem of estimating rates from noisy or censored Poisson counting data arises in various subdisciplines of physics. In astrophysics, for example, the counts are photons emitted from a distant source. In particle accelerator experiments, the counts are indirect measurements of the number of particles produced by a high-energy collision. In such contexts the observed counts typically include some additive background noise, such as ambient particles. In many cases there is also multiplicative noise, caused, for example, by photon censoring or particle decay, which further complicates the process of estimating the rate of interest.

 $n \sim Pois(\varepsilon \cdot s + b)$   $y \sim Pois(t \cdot b)$  $z \sim Pois(u \cdot \varepsilon)$ 

Edlefsen Liu, Dempster, "Estimating Limits from Poisson counting data using Dempster–Shafer analysis", 2009 The Annals of Applied Statistics, Vol.3, 764, astro-ph 0812.1690

### Challenges Strong Lens Time Delay Challenge Hyungsuk Tak

![](_page_70_Figure_1.jpeg)

Tak et al 2014 Bayesian Approach to Time Delay Estimation