# FISH TALES

If you give a man a fish he will eat for a day. If you teach a man to fish he will eat for a lifetime.

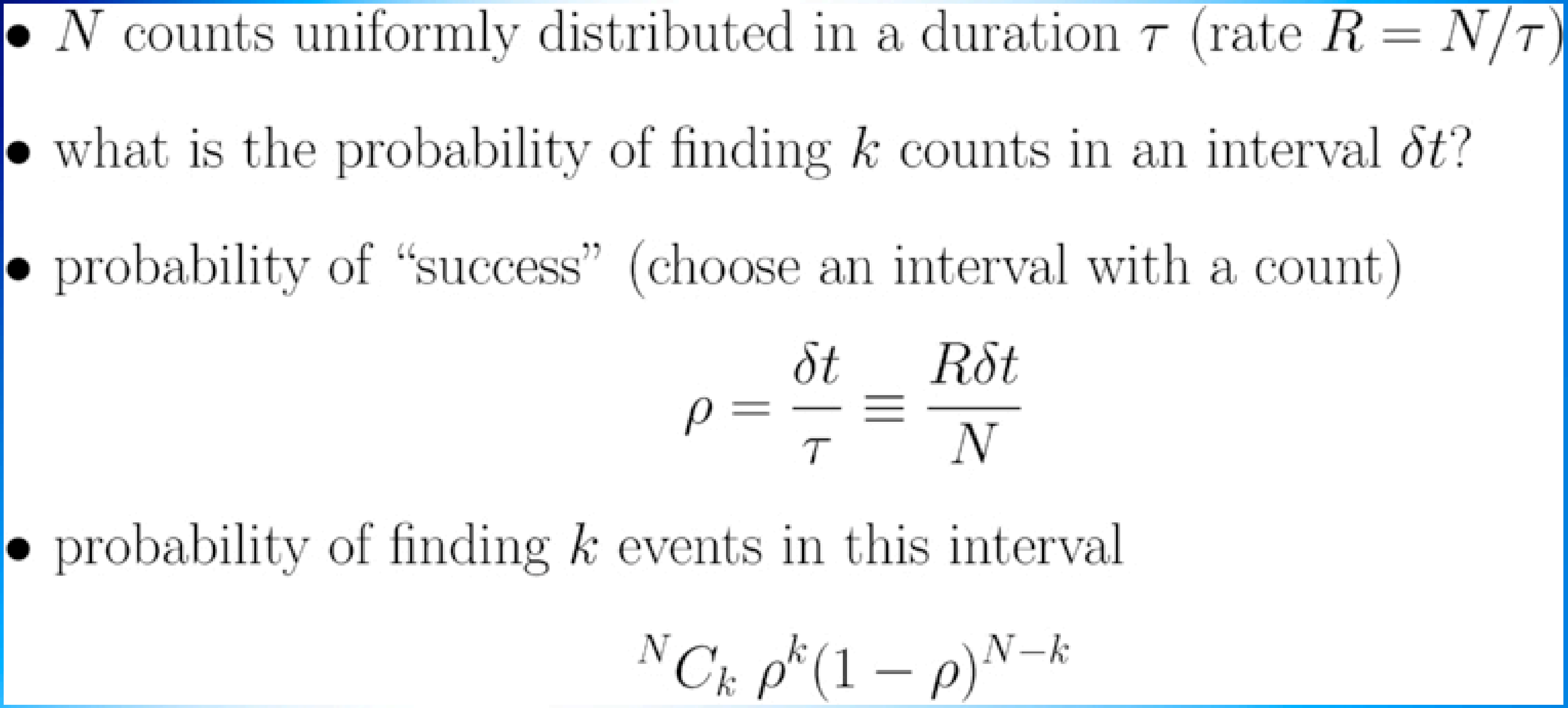
# POISSON TALES

If you give a man Cash he will analyze for a day. If you teach a man Poisson, he will analyze for a lifetime.

# It is all about Probabilities!

- The Poisson Likelihood
- Probability Calculus – Bayes’ Theorem
- Priors – the gamma distribution
- Source intensity and background marginalization
- Hardness Ratios (BEHR)

# Deriving the Poisson Likelihood



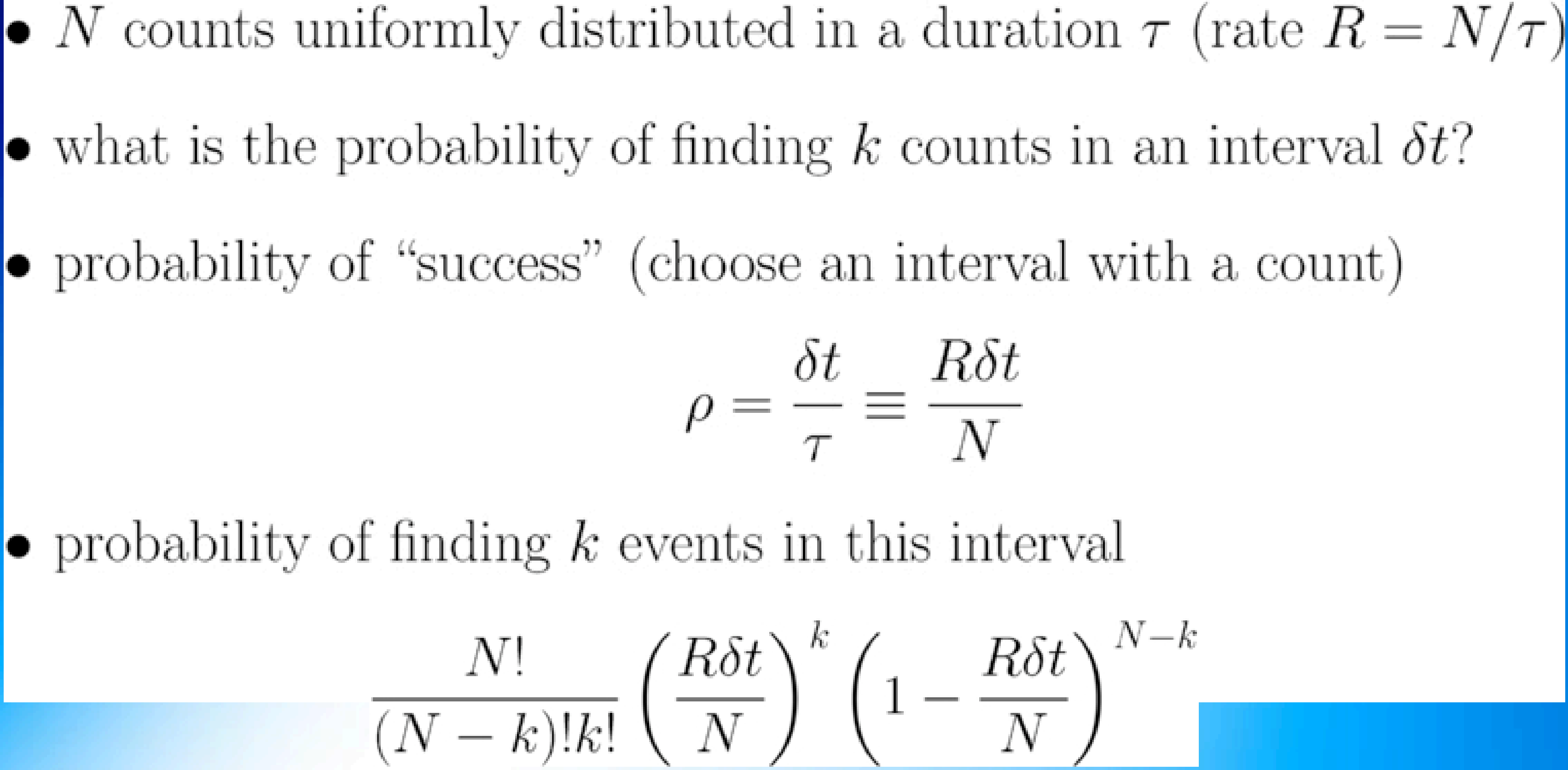
- $N$ counts uniformly distributed in a duration $\tau$ (rate $R = N/\tau$)
- what is the probability of finding $k$ counts in an interval $\delta t$?
- probability of "success" (choose an interval with a count)

$$\rho = \frac{\delta t}{\tau} \equiv \frac{R\delta t}{N}$$

- probability of finding $k$ events in this interval

$${}^{N}C_k \; \rho^k (1-\rho)^{N-k}$$

# Deriving the Poisson Likelihood



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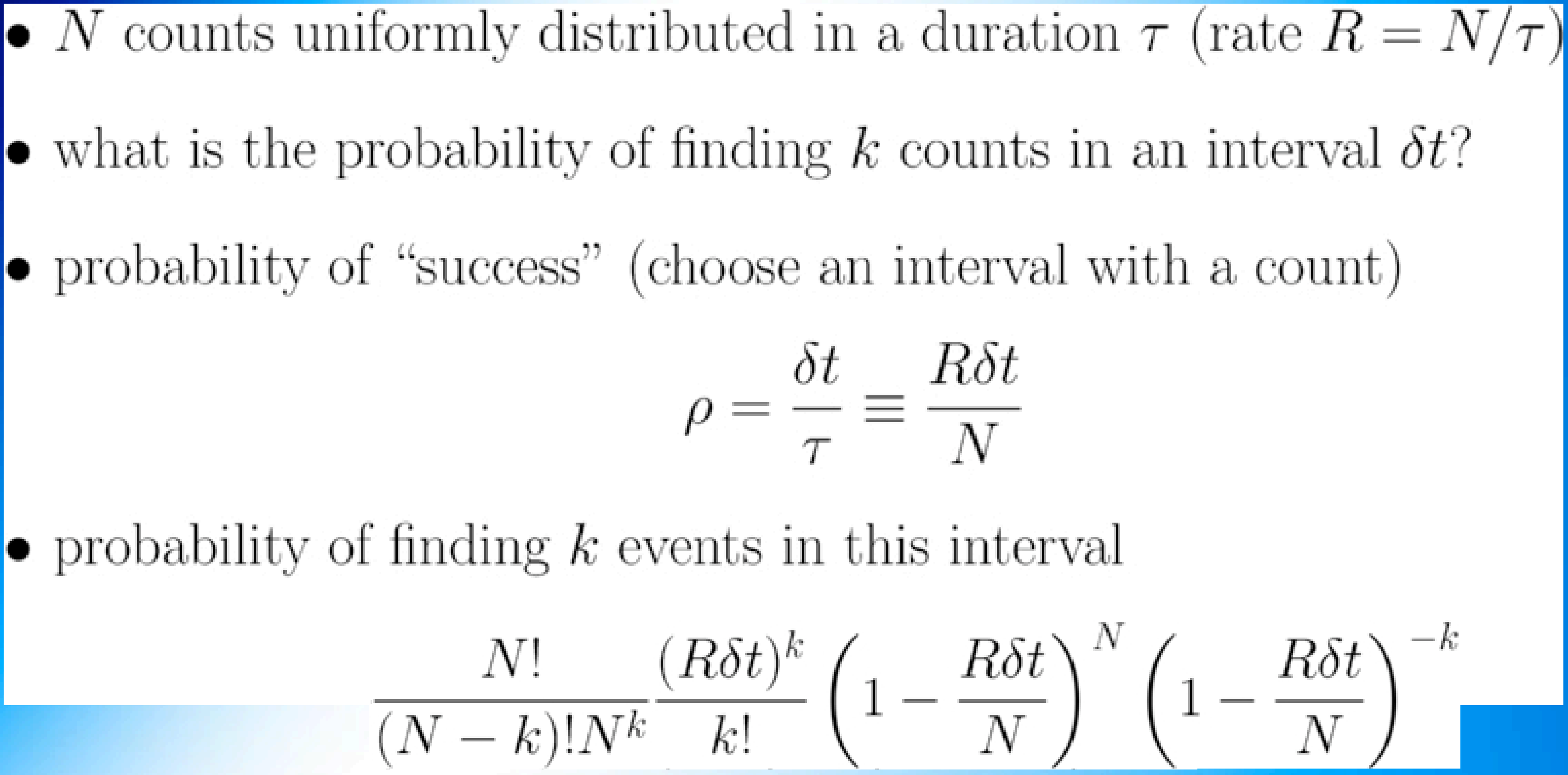
$$\frac{N!}{(N-k)!k!}\left(\frac{R\delta t}{N}\right)^{k}\left(1-\frac{R\delta t}{N}\right)^{N-k}$$

# Deriving the Poisson Likelihood



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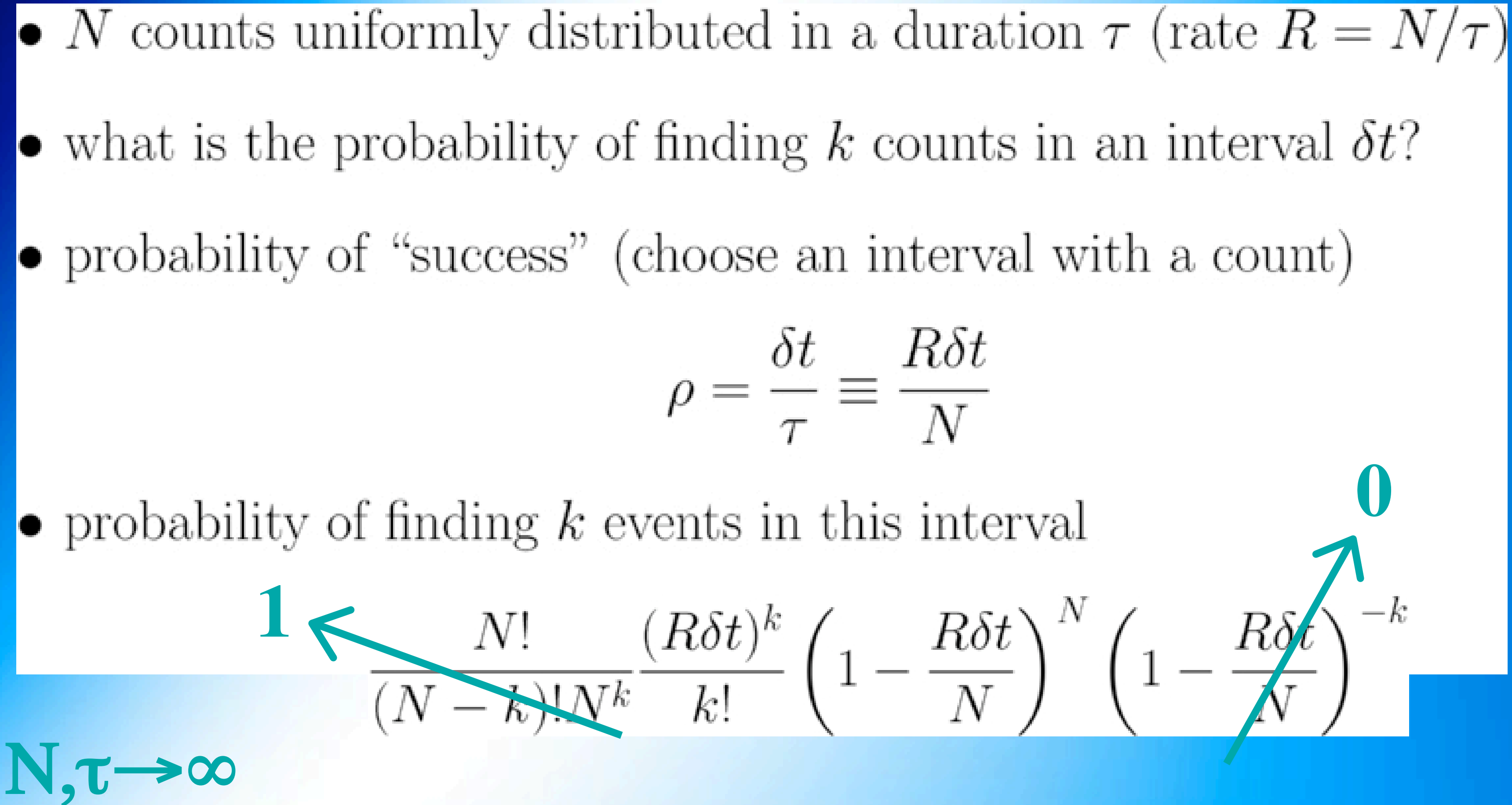
$$\frac{N!}{(N-k)!N^k}\frac{(R\delta t)^k}{k!}\left(1-\frac{R\delta t}{N}\right)^N\left(1-\frac{R\delta t}{N}\right)^{-k}$$

# Deriving the Poisson Likelihood



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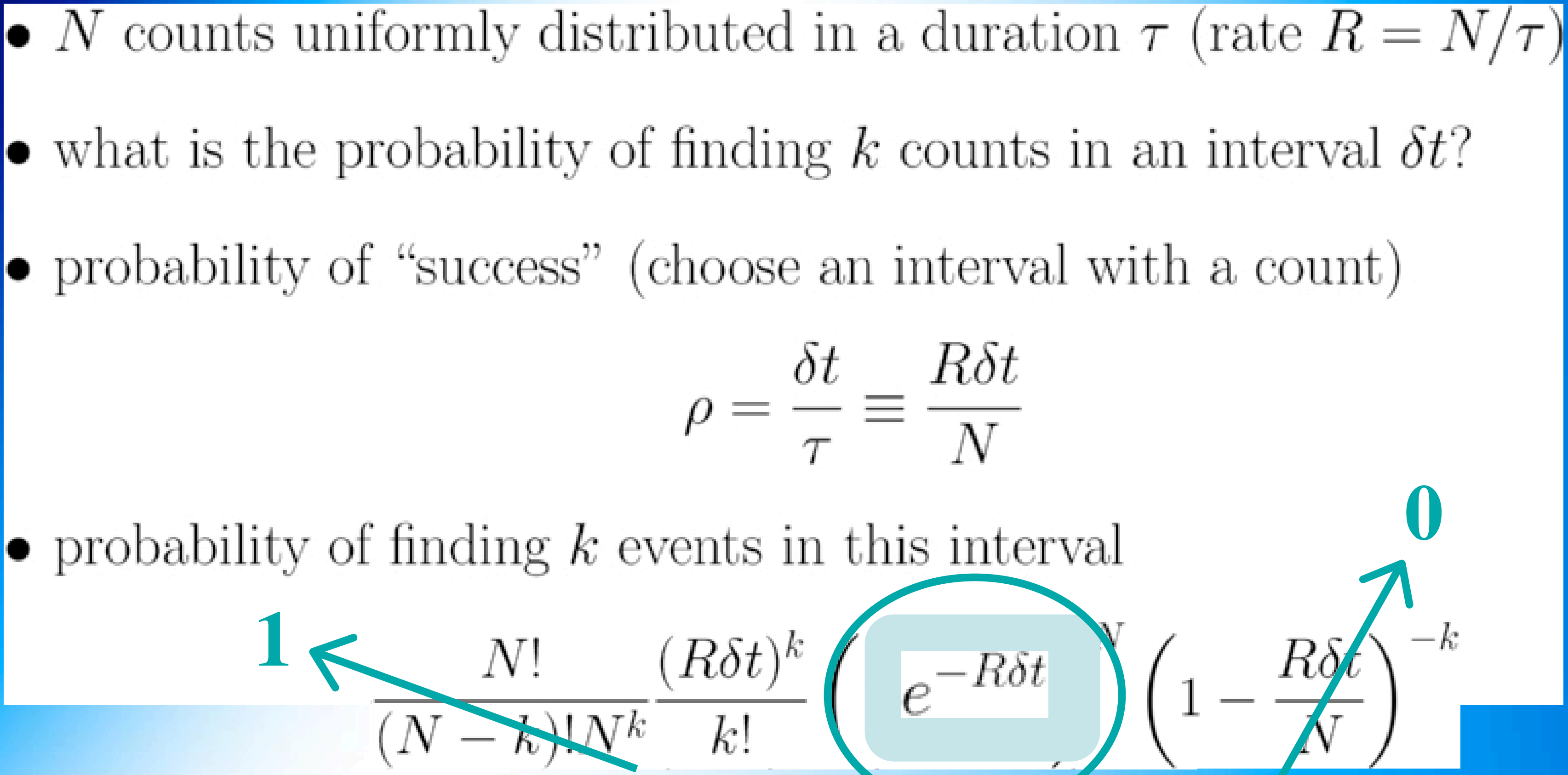
$N,\tau \to \infty$

# Deriving the Poisson Likelihood



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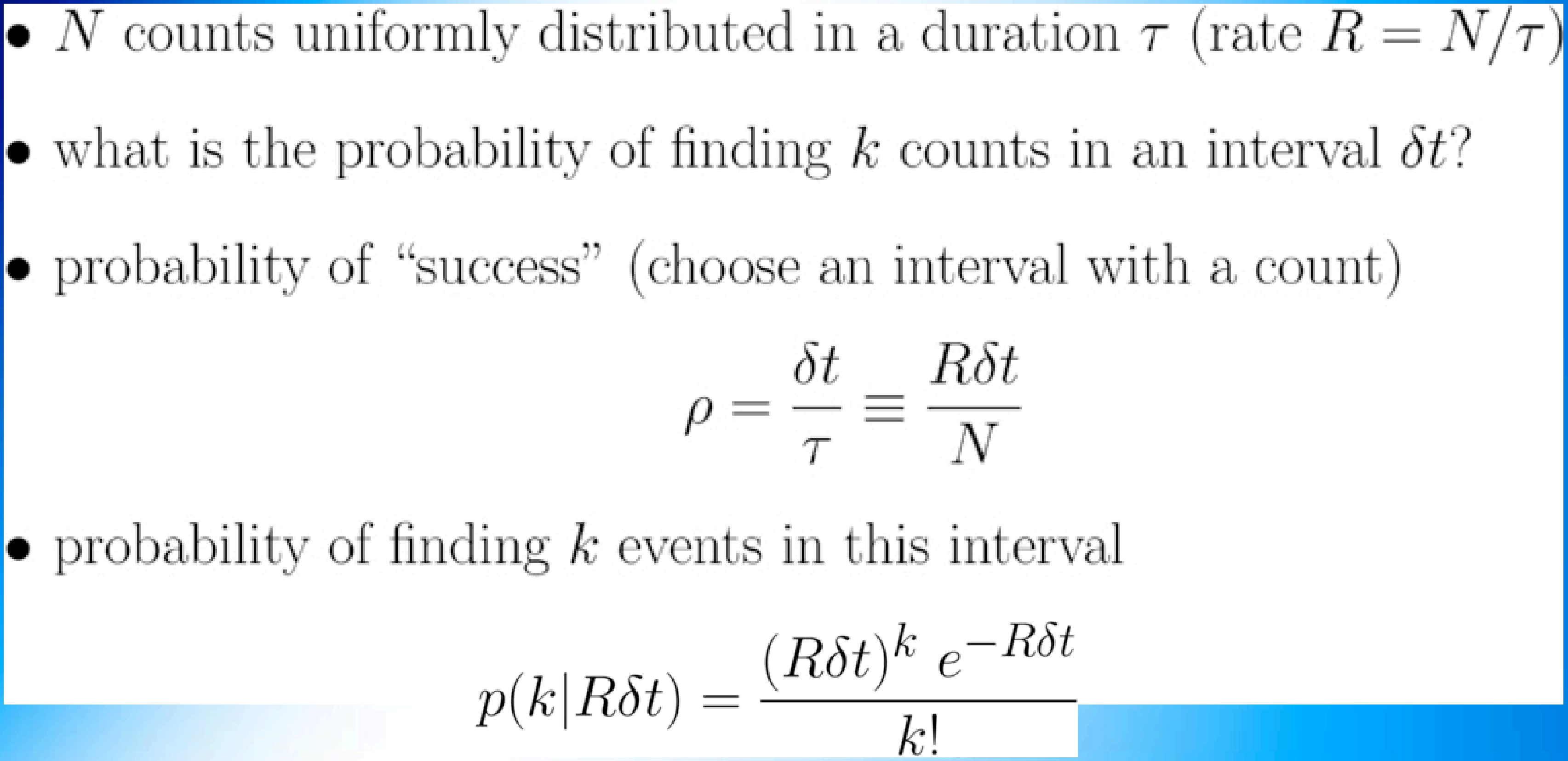
$$\underbrace{\frac{N!}{(N-k)!N^k}}_{1} \frac{(R\delta t)^k}{k!} \boxed{\left(e^{-R\delta t}\right)^N} \left(1 - \underbrace{\frac{R\delta t}{N}}_{0}\right)^{-k}$$

N,τ→∞

# Deriving the Poisson Likelihood



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- probability of finding $k$ events in this interval

$$p(k|R\delta t) = \frac{(R\delta t)^k \; e^{-R\delta t}}{k!}$$

# Probability Calculus

***A or B*** :: $p(A+B) = p(A) + p(B) - p(AB)$

***A and B*** :: $p(AB) = p(A|B)\ p(B) = p(B|A)\ p(A)$

***Bayes' Theorem*** :: $p(B|A) = p(A|B)\ p(B)\ /\ p(A)$

$$p(\text{Model}\ |\text{Data}) = p(\text{Model})\ p(\text{Data}|\text{Model})\ /\ p(\text{Data})$$

| | | | |
|---|---|---|---|
| posterior distribution | prior distribution | likelihood | normalization |

***marginalization*** :: $p(a|D) = \int p(ab|D)\ db$

# Priors

# Priors

- Incorporate known information
- Forced acknowledgement of bias
- Non-informative priors
  - flat
  - range
  - least informative (Jeffrey's)
  - gamma

# Priors

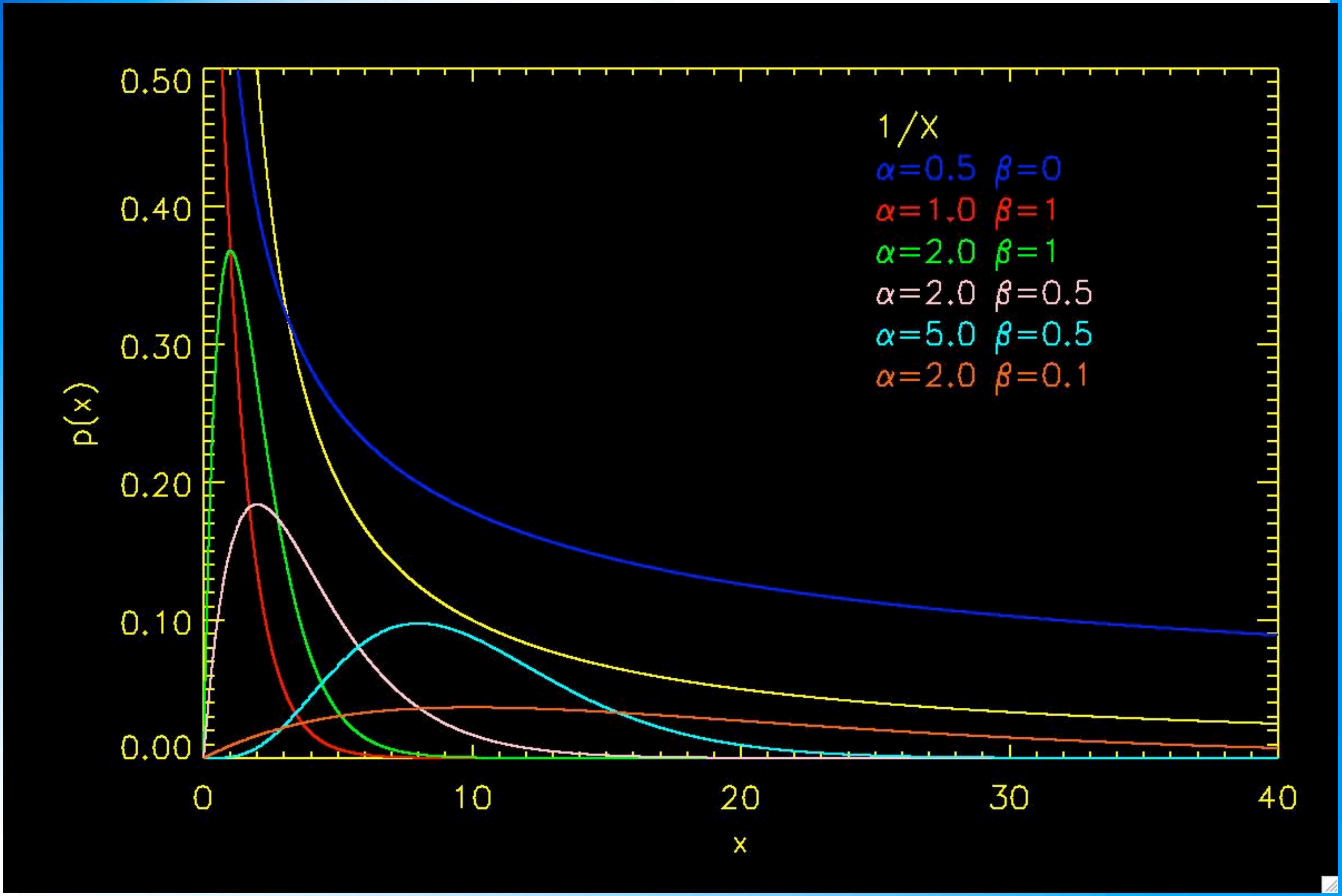
the gamma distribution

$$\gamma(x;\alpha,\beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$$

$$\overline{x} = \frac{\alpha}{\beta} \; , \quad \overline{x^2} - \overline{x}^2 = \frac{\alpha}{\beta^2}$$

# Priors

the gamma distribution $\gamma(x;\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$



# Panning for Gold:
# Source and Background

$$p(b|N_BI) = \frac{p(b|I)p(N_B|bI)}{p(N_B|I)}$$

$$p(b|N_BI) = \frac{1}{p(N_B|I)}\frac{\beta_B^{\alpha_B}\left(\frac{a_B}{a_S}\right)^{N_B}}{\Gamma(\alpha_B)\Gamma(N_B+1)}b^{N_B+\alpha_B-1}e^{-b\left(\beta_B+\frac{a_B}{a_S}\right)}$$

# Panning for Gold: Source and Background

$$p(b|N_B I) = \frac{p(b|I)p(N_B|bI)}{p(N_B|I)}$$

$$p(s|N_S I) = \int d\,b\; p(sb|N_S I)$$

$$p(sb|N_S I) = \frac{p(b|N_B I)p(s|I)p(N_S|sbI)}{p(N_S|I)}$$

$$p(sb|N_S I) = \frac{(\beta_B + r)^{\alpha_B + N_B}\beta_S^{\alpha_S}}{\Gamma(\alpha_B + N_B)\Gamma(\alpha_S)} \frac{b^{\alpha_B + N_B - 1} s^{\alpha_S - 1} (s + b)^{N_S}}{\Gamma(N_S + 1)} \frac{e^{-b(1+(\beta_B + r))} e^{-s(1+\beta_S)}}{p(N_S|I)}$$

# Panning for Gold: Source and Background

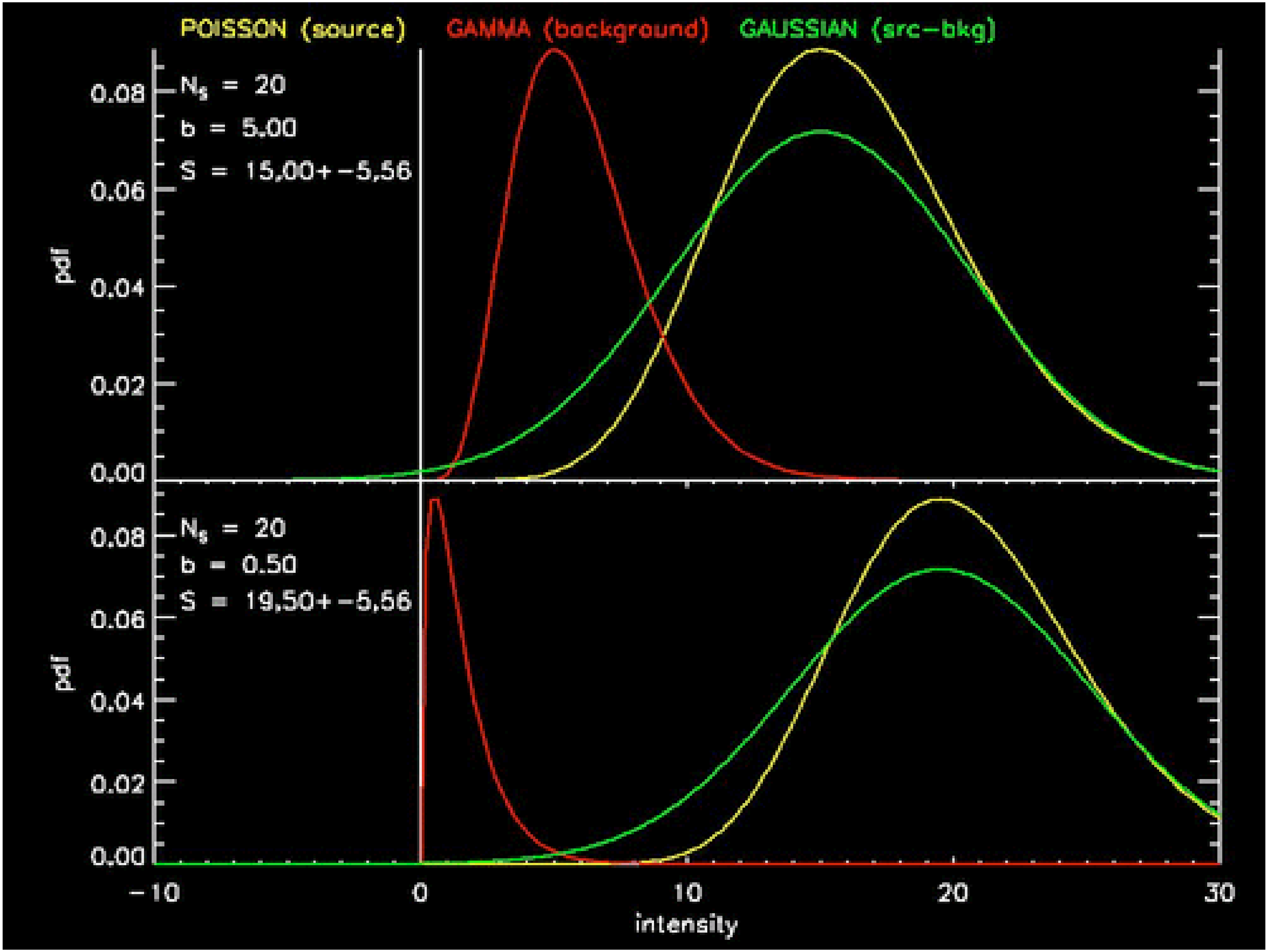
$$p(b|N_B I) = \frac{p(b|I)p(N_B|bI)}{p(N_B|I)}$$

$$p(s|N_S I) = \int db\; p(sb|N_S I)$$

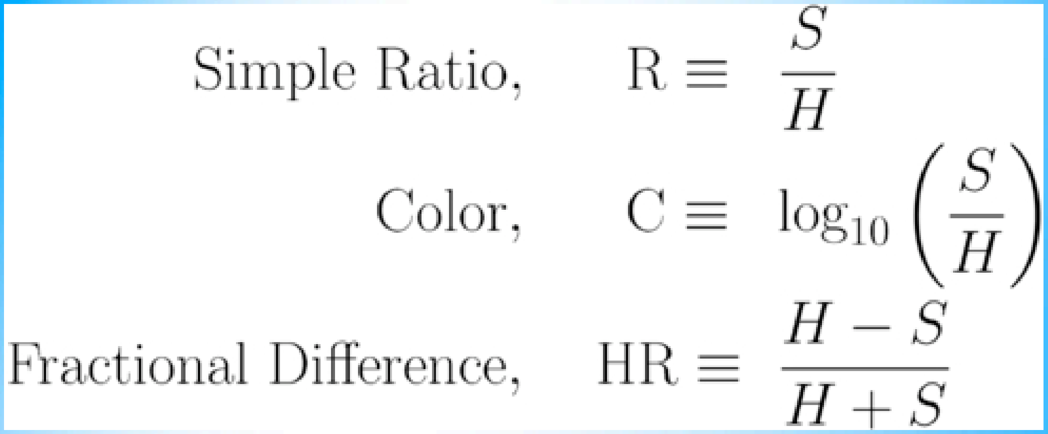
$$p(sb|N_S I) = \frac{p(b|N_B I)p(s|I)p(N_S|sbI)}{p(N_S|I)}$$

$$p(s|N_S I) = \left(\frac{1}{\sum_{k=0}^{N_S} \mathcal{I}_k^{bs}}\right) e^{-s(1+\beta)} \sum_{k=0}^{N_S} \frac{\Gamma(\alpha_B + N_B + N_S - k)}{\Gamma(k+1)\Gamma(N_S - k + 1)} \frac{s^{k+\alpha_S-1}}{(\beta_B + r + 1)^{\alpha_B + N_B + N_S - k}}$$

$$\mathcal{I}_k^{bs} = \frac{\Gamma(\alpha_B + N_B + N_S - k)\Gamma(k + \alpha_S)}{\Gamma(k+1)\Gamma(N_S - k + 1)} \frac{1}{(\beta_B + r + 1)^{\alpha_B + N_B + N_S - k}(1 + \beta_S)^{k+\alpha_S}}$$


POISSON (source)
GAMMA (background)
GAUSSIAN (src−bkg)
$N_s$ = 20
b = 5.00
S = 15.00+−5.56
$N_s$ = 20
b = 0.50
S = 19.50+−5.56
pdf
pdf
0.08
0.06
0.04
0.02
0.00
0.08
0.06
0.04
0.02
0.00
−10
0
10
20
30
intensity

# Hardness Ratios



$$\text{Simple Ratio,} \qquad \mathrm{R} \equiv \frac{S}{H}$$

$$\text{Color,} \qquad \mathrm{C} \equiv \log_{10}\left(\frac{S}{H}\right)$$

$$\text{Fractional Difference,} \qquad \mathrm{HR} \equiv \frac{H-S}{H+S}$$

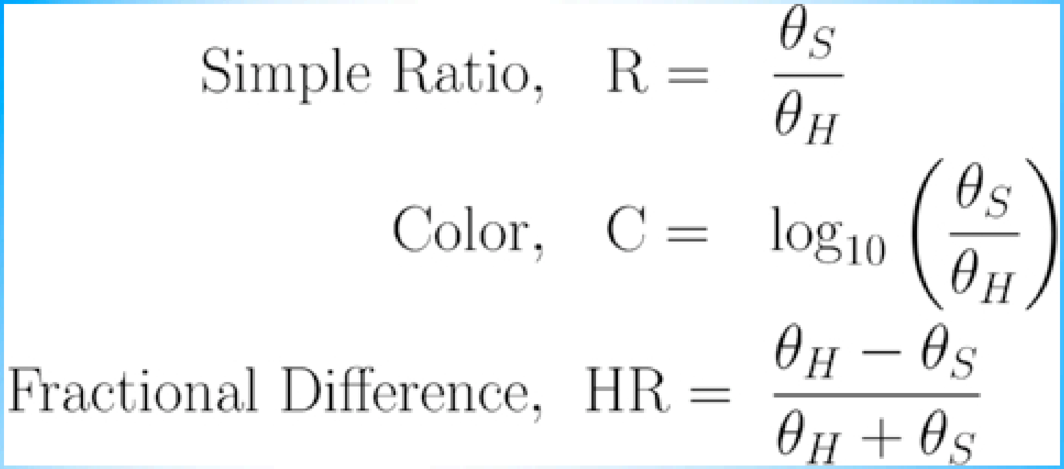
Simple, robust, intuitive summary

Proxy for spectral fitting

Useful for large samples

Most needed for low counts

# Hardness Ratios



$$\text{Simple Ratio,} \quad \mathrm{R} = \frac{\theta_S}{\theta_H}$$

$$\text{Color,} \quad \mathrm{C} = \log_{10}\left(\frac{\theta_S}{\theta_H}\right)$$

$$\text{Fractional Difference,} \quad \mathrm{HR} = \frac{\theta_H - \theta_S}{\theta_H + \theta_S}$$

Simple, robust, intuitive summary

Proxy for spectral fitting

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Most needed for low counts

$$\mathrm{R} = \theta_S/\theta_H \ ,$$

$$p(\mathrm{R}, \theta_H | S, H, B_S, B_H) \, d\mathrm{R} \, d\theta_H$$

$$= \ p(\theta_S, \theta_H | S, H, B_S, B_H) \Big| \frac{\partial(\theta_S, \theta_H)}{\partial(\mathrm{R}, \theta_H)} \Big| \, d\theta_S \, d\theta_H$$

$$= \ p(\mathrm{R}\theta_H, \theta_H | S, H, B_S, B_H) \theta_H \, d\mathrm{R} \, d\theta_H$$

$$\mathrm{C} = \log_{10}(\theta_S/\theta_H) \ ,$$

$$p(\mathrm{C}, \theta_H | S, H, B_S, B_H) \, d\mathrm{C} \, d\theta_H$$

$$= \ p(\theta_S, \theta_H | S, H, B_S, B_H) \Big| \frac{\partial(\theta_S, \theta_H)}{\partial(\mathrm{C}, \theta_H)} \Big| \, d\theta_S \, d\theta_H$$

$$= \ p(10^{\mathrm{C}}\theta_H, \theta_H | S, H, B_S, B_H) 10^{\mathrm{C}} \ln(10) \theta_H \, d\mathrm{C} \, d\theta_H$$

$$\mathrm{HR} = (\theta_H - \theta_S)/\omega \ ,$$

$$\omega = \theta_S + \theta_H \ ,$$

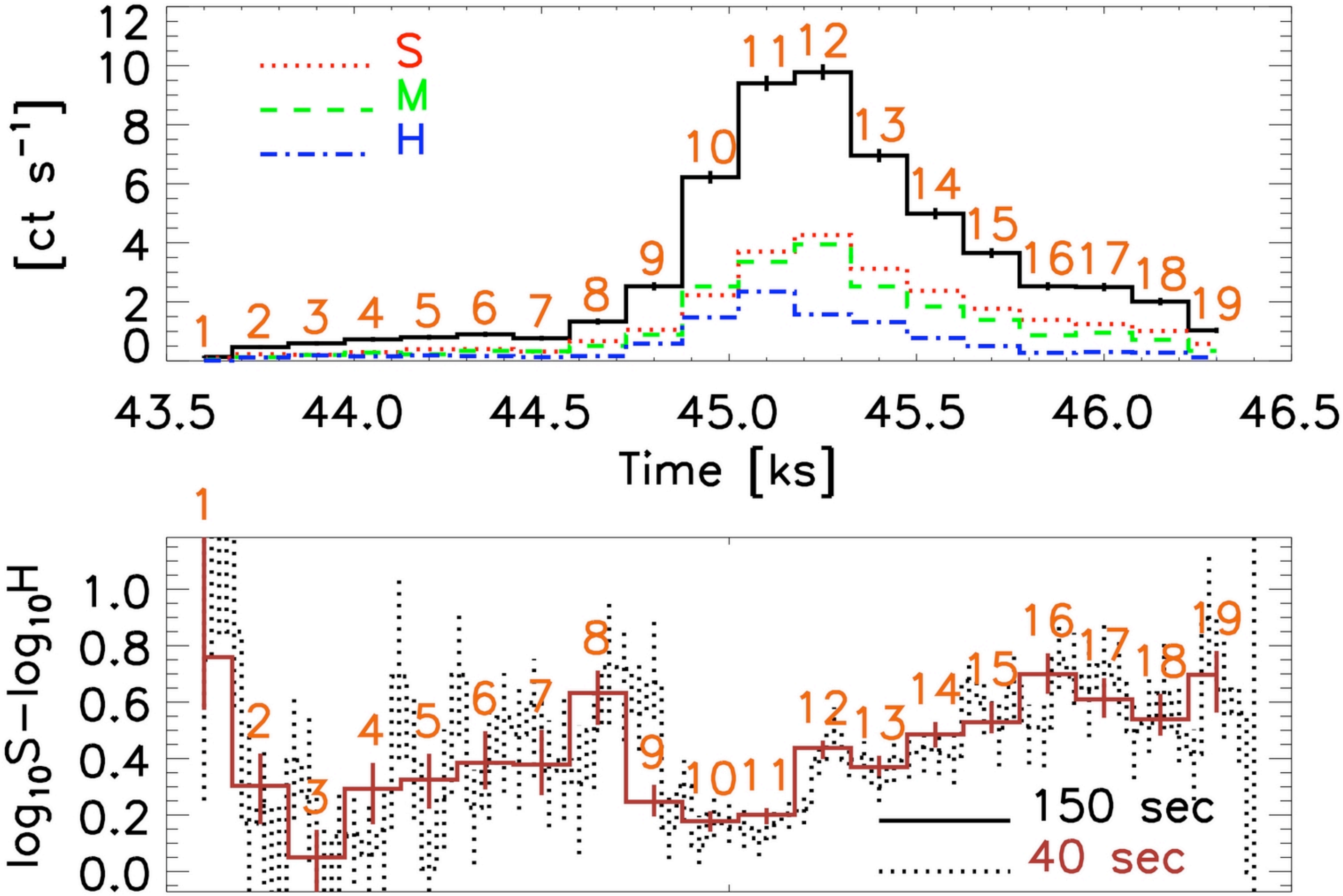
$$p(\mathrm{HR}, \omega | S, H, B_S, B_H) \, d\mathrm{HR} \, d\omega$$

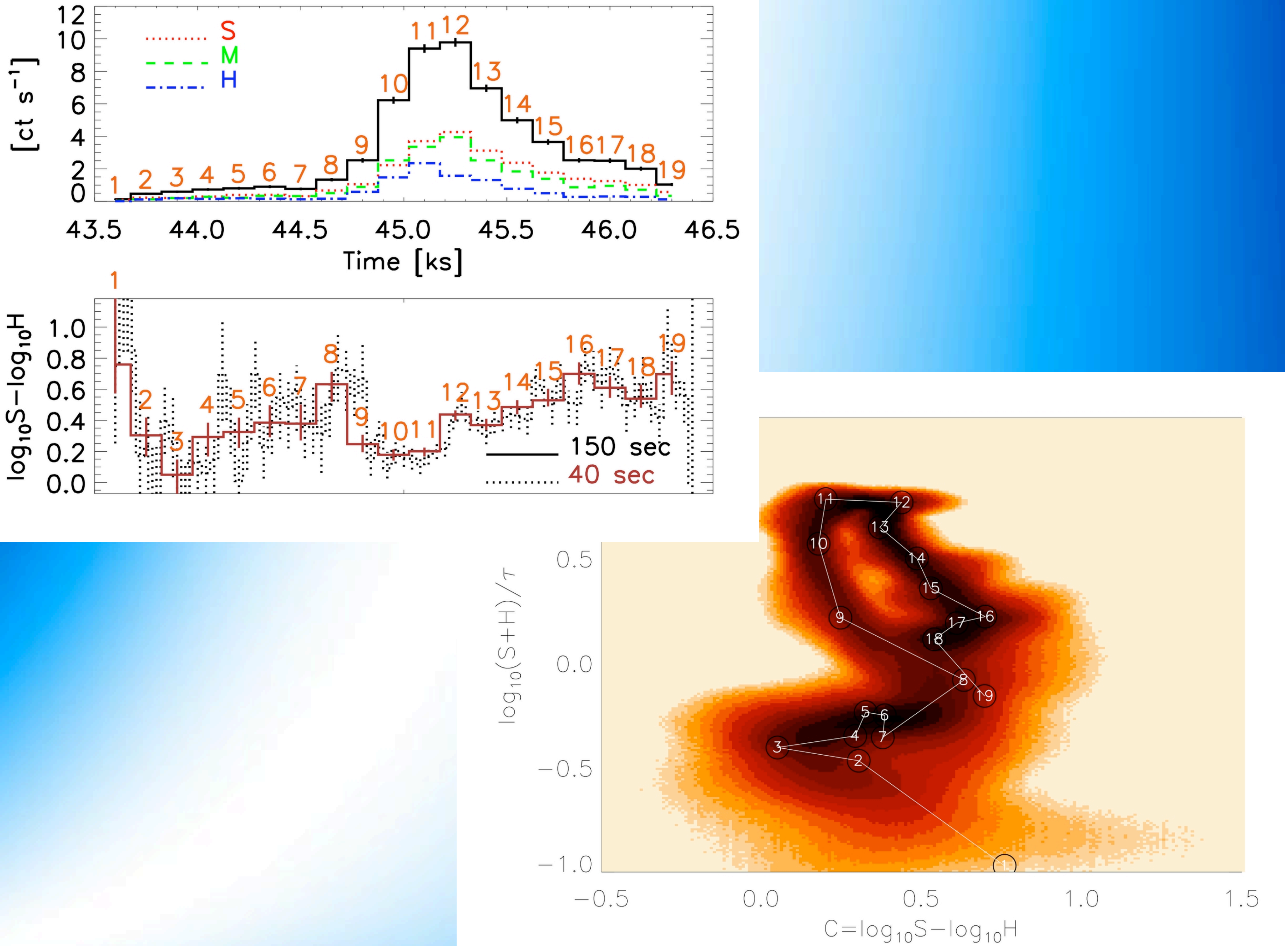
$$= \ p(\theta_S, \theta_H | S, H, B_S, B_H) \Big| \frac{\partial(\theta_S, \theta_H)}{\partial(\mathrm{HR}, \omega)} \Big| \, d\theta_S \, d\theta_H$$

$$= \ p(\frac{(1 - \mathrm{HR})\omega}{2}, \frac{(1 + \mathrm{HR})\omega}{2} | S, H, B_S, B_H) \frac{\omega}{2} \, d\mathrm{HR} \, d\omega$$

# *BEHR*

**http://hea-www.harvard.edu/AstroStat/BEHR/**





[ct s$^{-1}$]
0
2
4
6
8
10
12
S
M
H
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
43.5
44.0
44.5
45.0
45.5
46.0
46.5
Time [ks]
$\log_{10}S - \log_{10}H$
0.0
0.2
0.4
0.6
0.8
1.0
150 sec
40 sec
$\log_{10}(S+H)/\tau$
0.5
0.0
−0.5
−1.0
−0.5
0.0
0.5
1.0
1.5
$C = \log_{10}S - \log_{10}H$